Choosing a period total fertility index using a simulation model

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Abstracts and PDF of the PowerPoint presentation.

Short abstract

In the last fifteen years, there has been a flurry of proposals made for accurately measuring total period fertility, in order to improve the results given by the classical Total Fertility Rate (TFR). The main argument is that the TFR gives a biased value for the mean number of children women or men have, based on births that are observed in the current year. These proposals are either new ways to calculate a total period fertility level or procedures that intend to correct the biases existing total indexes have. The problem is that these additions to the demographer's toolset are not accompanied by the definition of criteria, which could allow one to choose the best fertility measure to use, depending on the kind of data available for each country.

Our objective is to apply a macro demographic simulation model in order to help analysts to choose the best or the less imperfect period fertility measure, depending on the data at hand and on a study of the evolution in time of the structure of age fertility rates. The results of the simulation model will be comparisons of the fertility levels obtained with the battery of available period indexes or correction methods, with the 'true' level entered as an input, in various scenarios of changing tempo, from simple to more complex one.

Extended Abstract

There is a situation of impending crisis in fertility analysis, as demographers have a large choice of competing fertility indexes or procedures for measuring the 'true' number of children men or women have in the current period, but at the same time, they have no clear criterion to decide which one is the best or, at least, is the most appropriate giving the kind of data available. It has long being recognized that the traditional Total Fertility Rate (TFR) is not really adequate for the study of fertility behaviour, as it gives a biased value for the period mean number of children. Authors like Hajnal (1947), Henry (1953) and Ryder (1964) have clearly established since the 1940s that when ages at childbearing are changing, because people advance, postpone or change the pace of their reproductive life cycle, the fertility level as measured by the TFR systematically deviates from the 'real' mean number of children people have. A classic solution to this problem has been to compute cohort indicators levels like the Cohort Fertility Rate (CFR) and to compare them with the TFR using an accounting scheme known as the "demographic translation". Doing this, the TFR is compared with what is supposed to be the 'true' fertility level, which corresponds to a cohort one.

This solution had been criticised for various reasons and from various angles. On the theoretical side, authors like Lee (1980), Pullum (1980), Ni Brochlain (1992) have argued that fertility behaviours are not cohort specific, and there is no sense in comparing the TFR with the CFR, because the difference in cohort fertility levels are not related to differences in cohort behaviours, but can be explained by chance factors: the proportion of 'good' or 'bad' years lived by each cohort during their reproductive life. On the practical side, using cohort indicators is highly problematic as demographers are forced to make strong assumptions on future fertility rates if they want to compute the cohort fertility levels associated to births occurring in the last observed time period. The main part of these births corresponds to women aged between 22 and 32 years. So in order to estimate the CFR, one has to complete the fertility for cohorts until they reach around 40 years of age, which casts a strong doubt on the results and whether they are the 'true' fertility levels.

The solutions proposed by a growing numbers of authors are to try to solve directly the problem of measuring period fertility, without a detour by a cohort accounting scheme. These solutions can be classified into two groups:

- Proposals for new way to calculate a total fertility index based on probabilistic kind of rates, extending the life table model to births, instead of using the traditional fertility rates which relate all births with all women. The latter kind of rates doesn't take into account that, for example, women already at parity one or more are not at risk of having birth of first order. These new indexes are based on the work of authors like Whelpton (1946) and Henry (1953), who proposed in the 1940s and 1950s the use of probabilistic rates and Henry even introduced a new class of fertility measure, the parity progression ratios that are direct precursors of the proposals for new indexes of recent authors like Rallu and Toulemon (1993).

- Proposals for correction methods intended to eliminate the effect of biases in existing total fertility indexes. For example Bongaarts and Feeney (1998) introduced a correction method to the TFR that uses only period data and provide a more accurate estimate of the 'true' period fertility level. Their correction had been on turn corrected and generalized by Kohler and Philipov (2001). Brass (1991) proposed a correction formula to Henry's parity progression ratio, based on rates by order and duration since the last birth. Kohler and Ortega (2002) developed a correction to parity progression ratios based on rates by parity and age.

In total, the new proposals, coupled with ancient ones that are revisited, offer more than ten ways to calculate a period total fertility level, with results that may differs by 20% or more. There is no general agreement on which method gives the best result, and on the robustness of the different methods to special cases of evolution in time of the structure of fertility rates.

We have developed a flexible macro simulation model, which allows us to calculate traditional fertility rate and rate of the probabilistic kind, by order, age and duration. The model uses a gamma function for the birth order and a beta one for the progression to second and third birth. We are able to model different evolutions in time of the tempo of fertility: linear change in time, change in the variance, and change in higher moments, with or without turning points. The idea is to model changes in time of the tempo of fertility, with a constant quantum, in order to bypass the problem of determining what is the 'true' level of fertility. Those rates are then used to calculate all the proposed total fertility indexes, and all the proposed correction to these indexes. The results may help in deciding which fertility indexes, corrected or not, are closer to the constant level entered as input, and which one are more robust in the situation of complex changes in the shape of fertility by age.

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Problems in Period Fertility Measurement (I)

- The Total Fertility Rate (TFR) is a *biased* indicator of the mean number of children when there are *tempo* changes (Ryder, 1964).
- What is the best way to estimate the *true* level of period fertility?
  - Use Occurrence-Exposure rates (Rates of first kind, or of probabilistic kind) instead of the classical Incidence rates (Rates of second kind) (Rallu & Toulemon, 1994).
  - Apply correction factors to the TFR in order to remove the effects of tempo changes (Bongaarts & Feeney, 1998).
  - Do both: use O-E rates based fertility indices *and* correct them for tempo changes (Brass, 1991; Kohler & Ortega, 2002).

Problems in Period Fertility Measurement (II)

- There are various ways to estimate Total Fertility using O-E rates, with no indication in the literature of which one is the best.
- We have analytical formulas describing the relationship between period TFR and cohort TFR, when there are tempo changes, *only* for some of the indices based on O-E rates. So there is *no general analytical* solution to the problem of choosing the best index, and we must use a *simulation* model in order to decide which index is the best.

Period indices of total fertility

Type of rate	Birth order	Index	Main dimensions	Author
Incidence	**All**	TFR	Age	Kuczynski (1928)
Occurrence Exposure	**1st**	PATFR	Age-Parity	Whelpton (1946)
	2nd and more	PATFR	Age-Parity	Whelpton (1946)
		PATFR_N	Age-Parity	Yamaguchi (2004)
		PDTFR	Duration-Parity	Henry (1953)
		PADTFR	Age-Duration-Parity	Rallu & Toulemon (1994)

Two types of fertility rates, and two types of sequence

Fertility index by birth order	Type of rate (by age and/or duration)
TFR(i)	$\frac{\text{Births of order i}}{\text{Total of women}}$
PATFR(i), PDTFR(i), PADTFR(i)	$\frac{\text{Births of order i}}{\text{Women at parity i-1}}$
PATFR_N(i)	$\frac{\text{Births of order i}}{\text{Women at parity 0, 1, …, i-1}}$

Analytical relationship between cohort and period indices with constant shift in age or duration schedule

Birth order	Index	Formula (for each order)	Author
All	TFR	CFR = TFR / (1 - r)	Bongaarts & Feeney (1998)
1st	PATFR	$CFR=1-(1-PATFR)^{(1/(1-r))}$	Brass (1991), Calot (1992), Keilman (1994), Zeng & Land (2002), Yamaguchi (2004)
2nd	PATFR_N	$CFR=1-(1-PATFR\_N)^{(1/(1-r))}$	
	PDTFR	$CFR=1-(1-PDTFR)^{(1/(1-d))}$	
	PATFR PADTFR	No simple analytical formula	

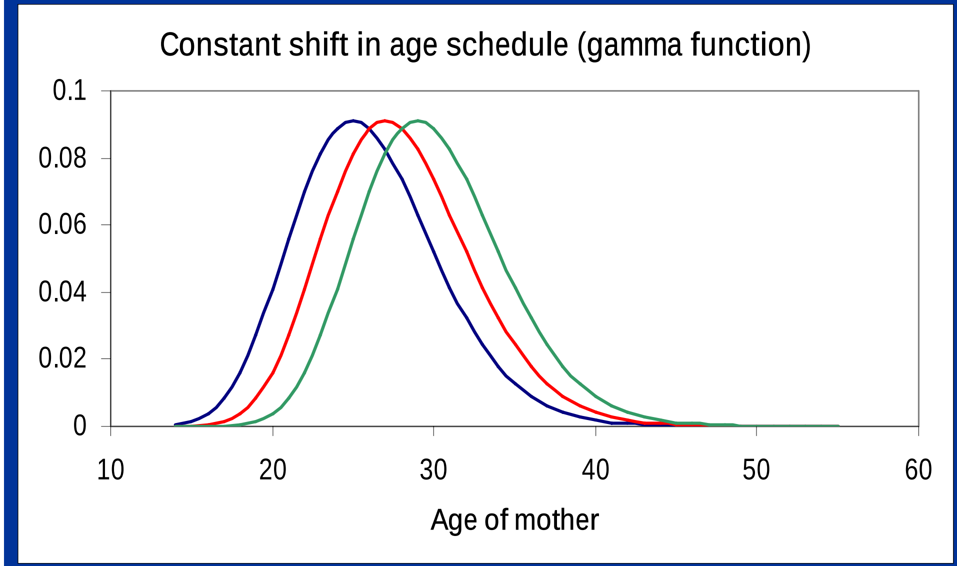
r is the yearly variation of the corresponding period mean age at *i*th childbearing
d is the equivalent variation for inter-birth intervals.

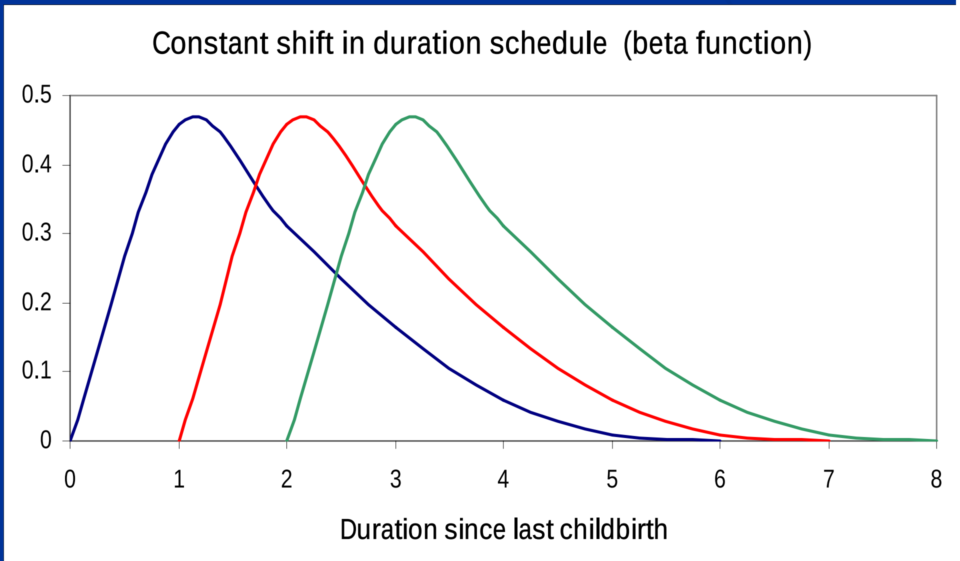
A simulation model for comparing period fertility indices (I)

- Fertility is simulated at the cohort and parity level.
- Fertility of order 1 is simulated using a *gamma* function which gives first births by age.
- Fertility of order 2 and 3 is simulated using a *beta* function for the interval between births, which gives second and third births by duration since the previous one. The duration schedule is shorter for higher ages.
- Both Incidence and Occurrence-Exposure (repeatable as well as non-repeatable) rates are computed.

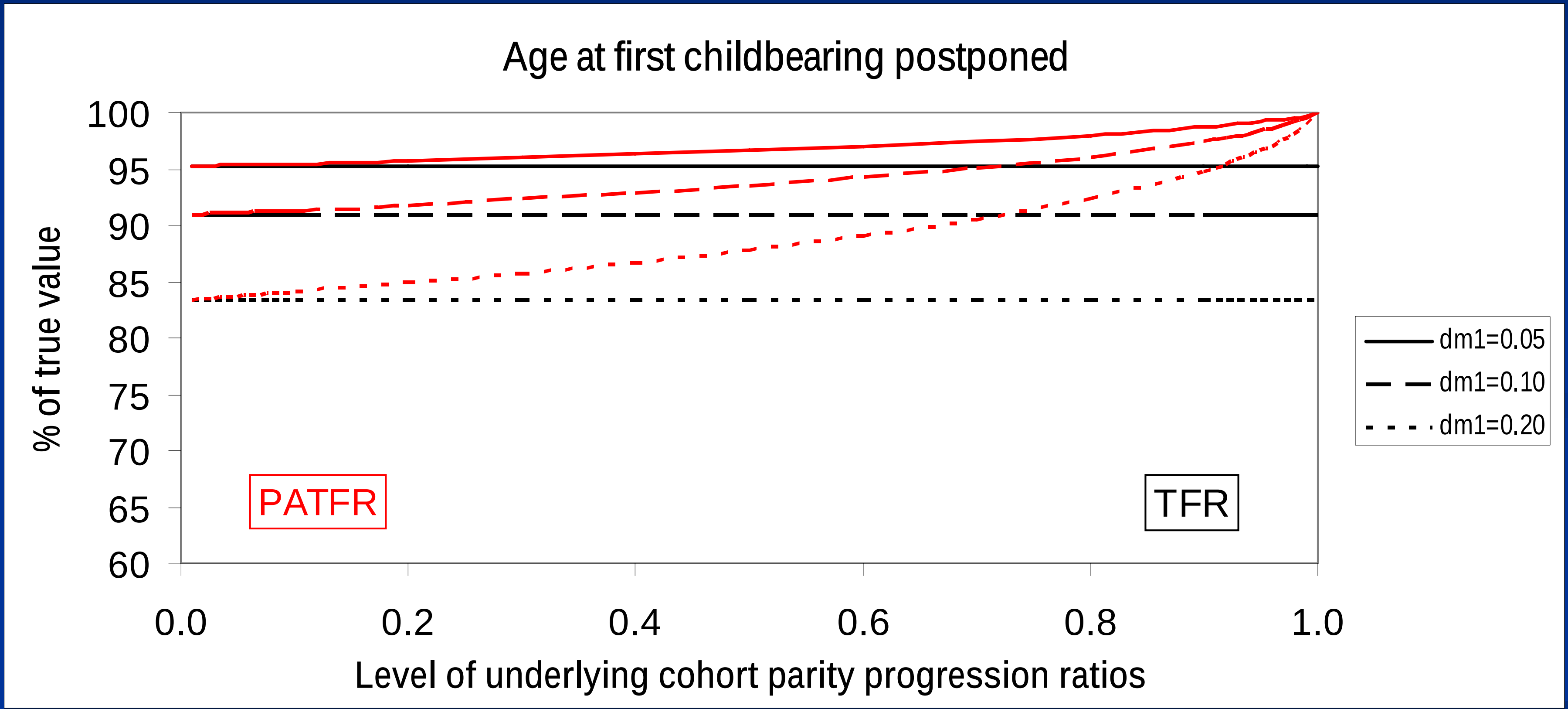
A simulation model for comparing period fertility indices (II)

Tempo changes are simulated by introducing a constant shift in age or duration schedule at the cohort level. *Quantum* level remains constant.



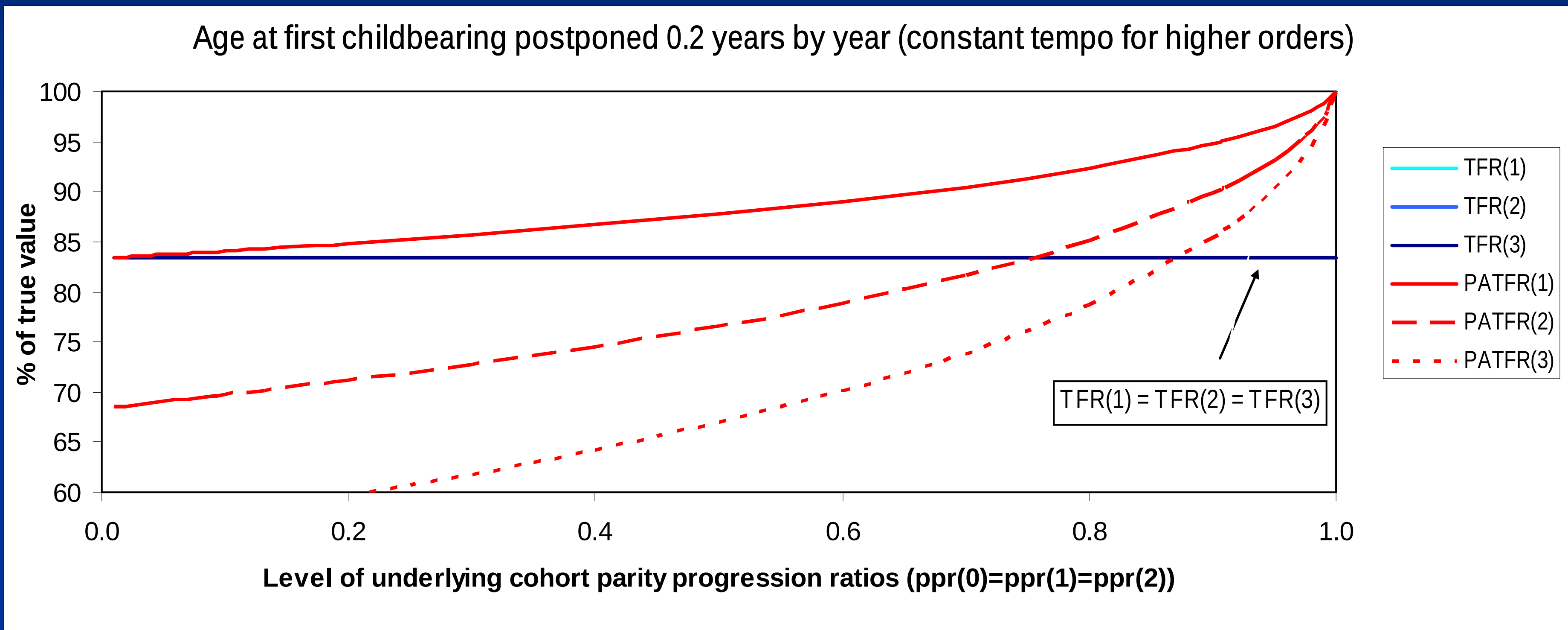


TFR(1) versus PATFR(1) (in proportion of cohort level)



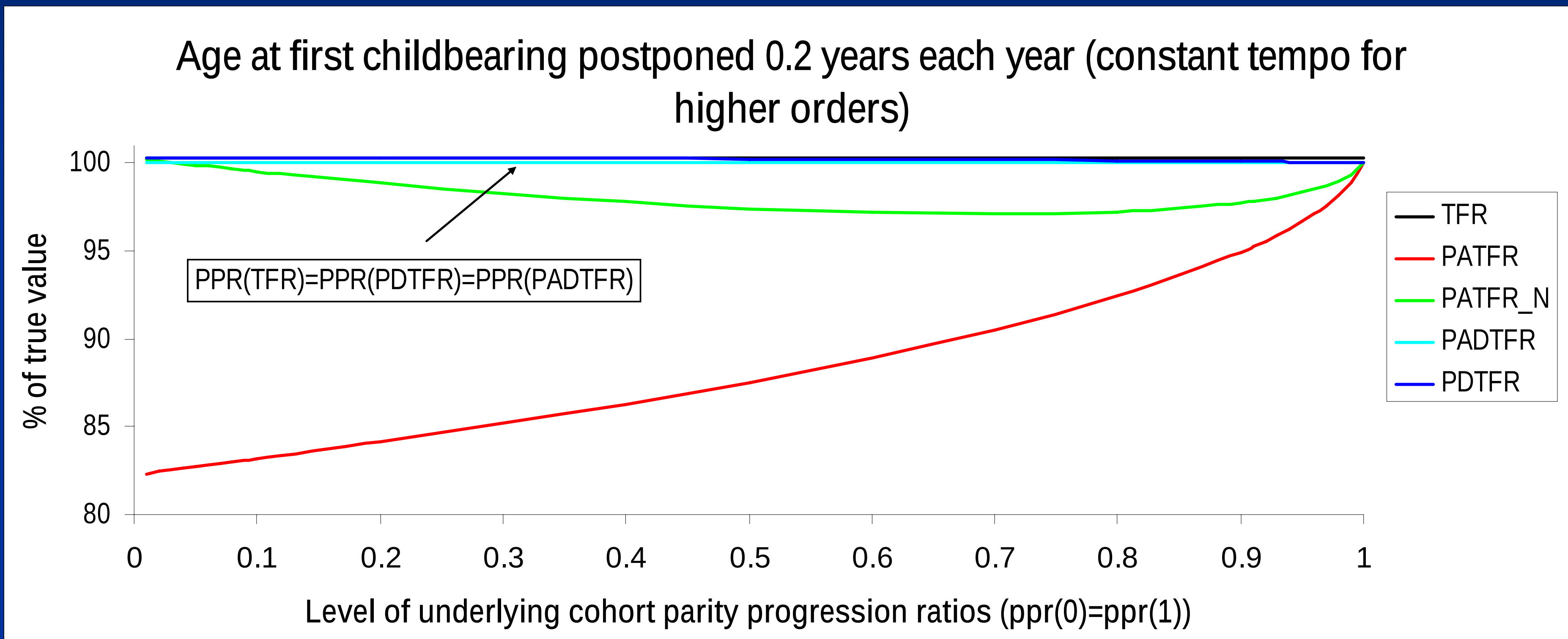
PATFR(1) is always closer to the cohort or *true* value than TFR(1). This is demonstrated analytically by Yamaguchi (2004)

TFR versus PATFR at 1st, 2nd & 3rd birth order (in proportion of cohort level)



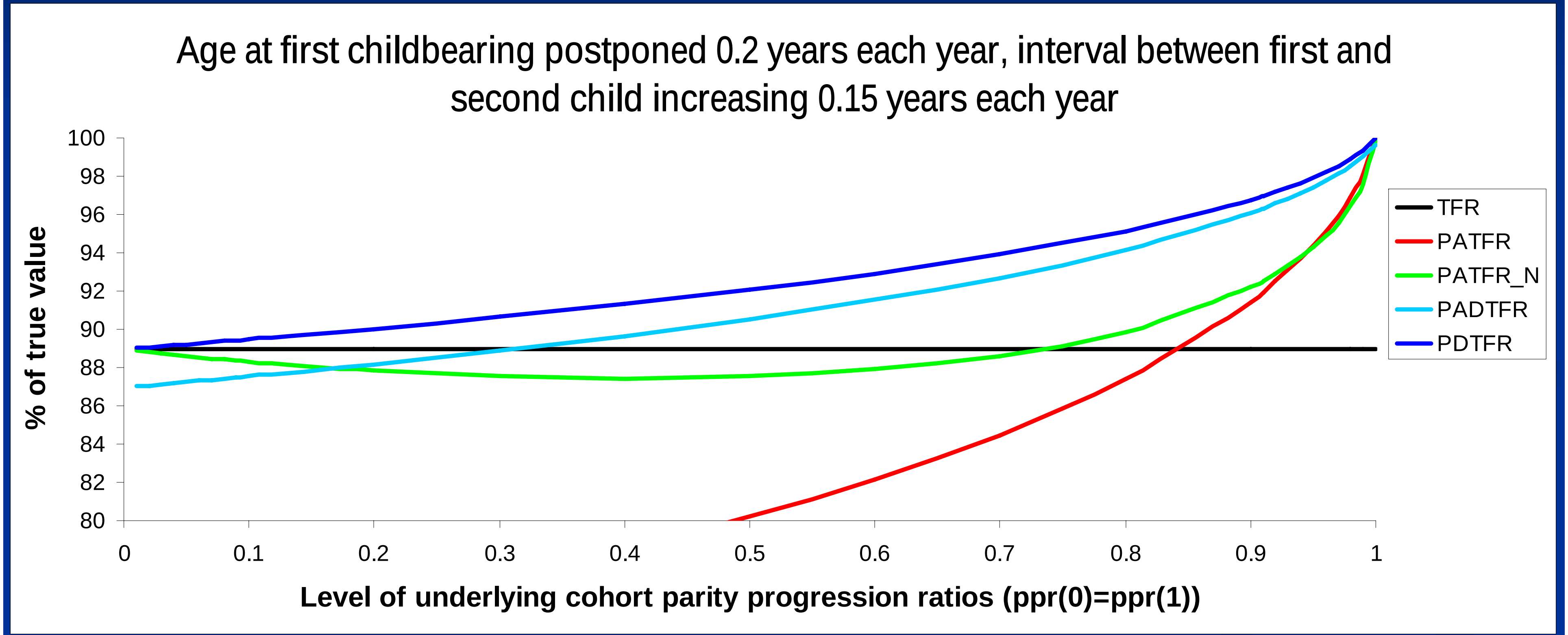
For birth orders 2 and more, the PATFR level is *strongly distorted* by change in tempo at the first order (astonishingly, nobody seems to have noted that before, or at least how much distortion there is!)

PPR computed on TFR vs on O-E rates based TFR, at order 2 (in proportion of cohort level) (I)



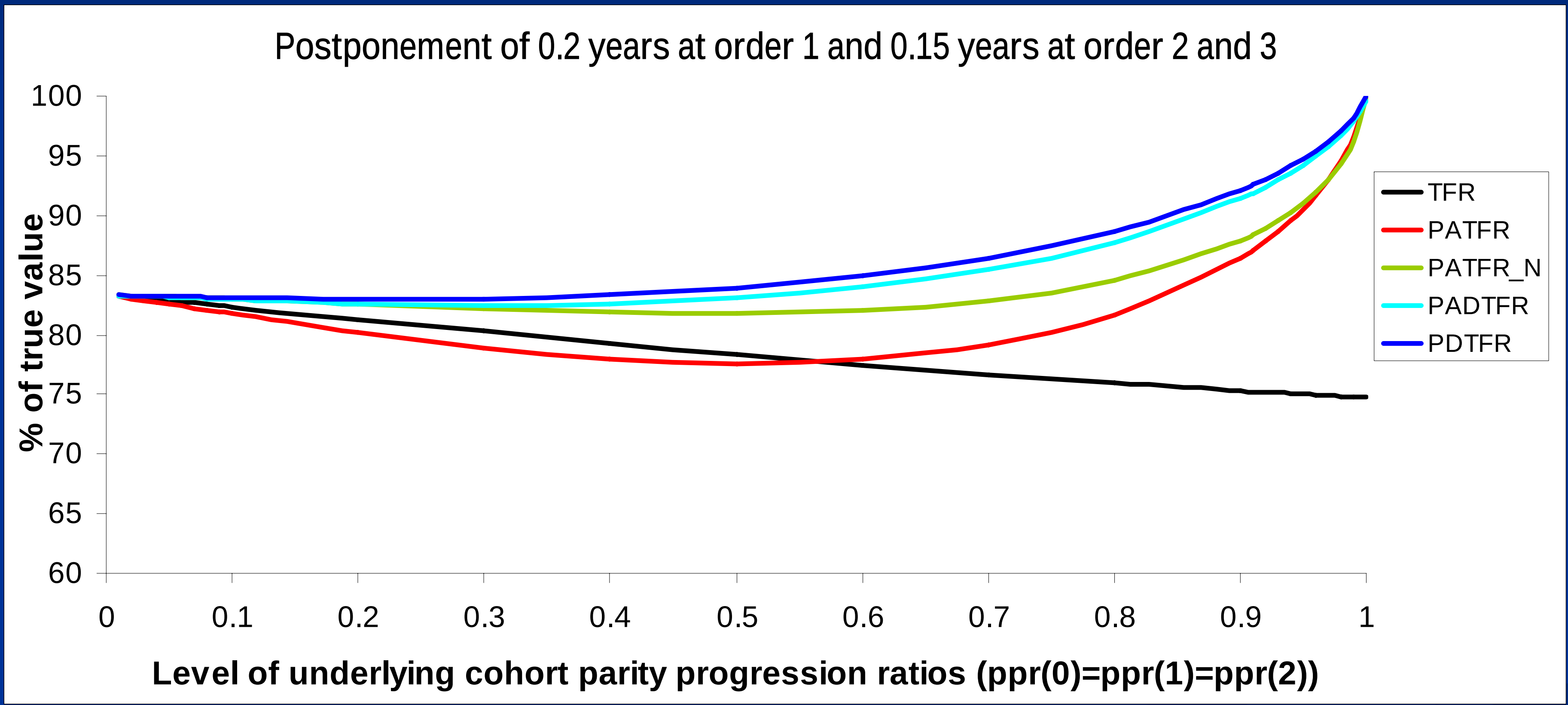
Only PDTFR and PADTFR (which include the inter-birth interval duration dimension) are not distorted by tempo changes at *previous* birth order.

PPR computed on TFR vs on O-E rates based TFR, at order 2 (in proportion of cohort level) (II)



All the indices are distorted by the tempo variation for the 2nd birth order, but PDTFR (based on intra-birth interval duration) is the best one when there are interactions effects between tempo changes at the age and the intra-birth interval duration levels.

Total Fertility (in proportion of cohort value)

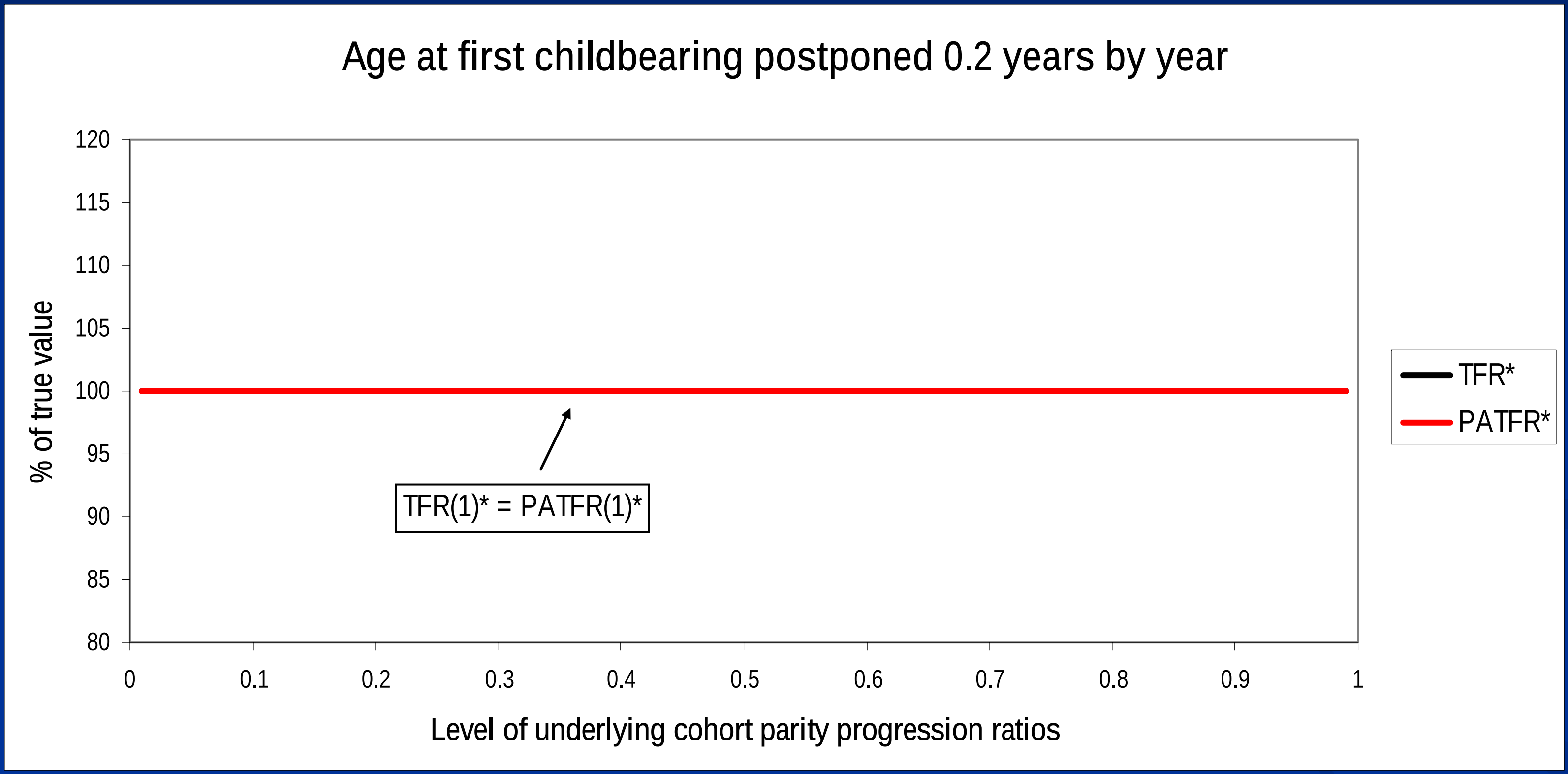


The fertility index based *only* on duration since the last event is always closer to the 'truth'. But every Total Fertility index *has to* be corrected from tempo distortion if we want to measure accurately the level of fertility.

Period Fertility indices based on Occurrence-Exposure rates need to be adjusted…

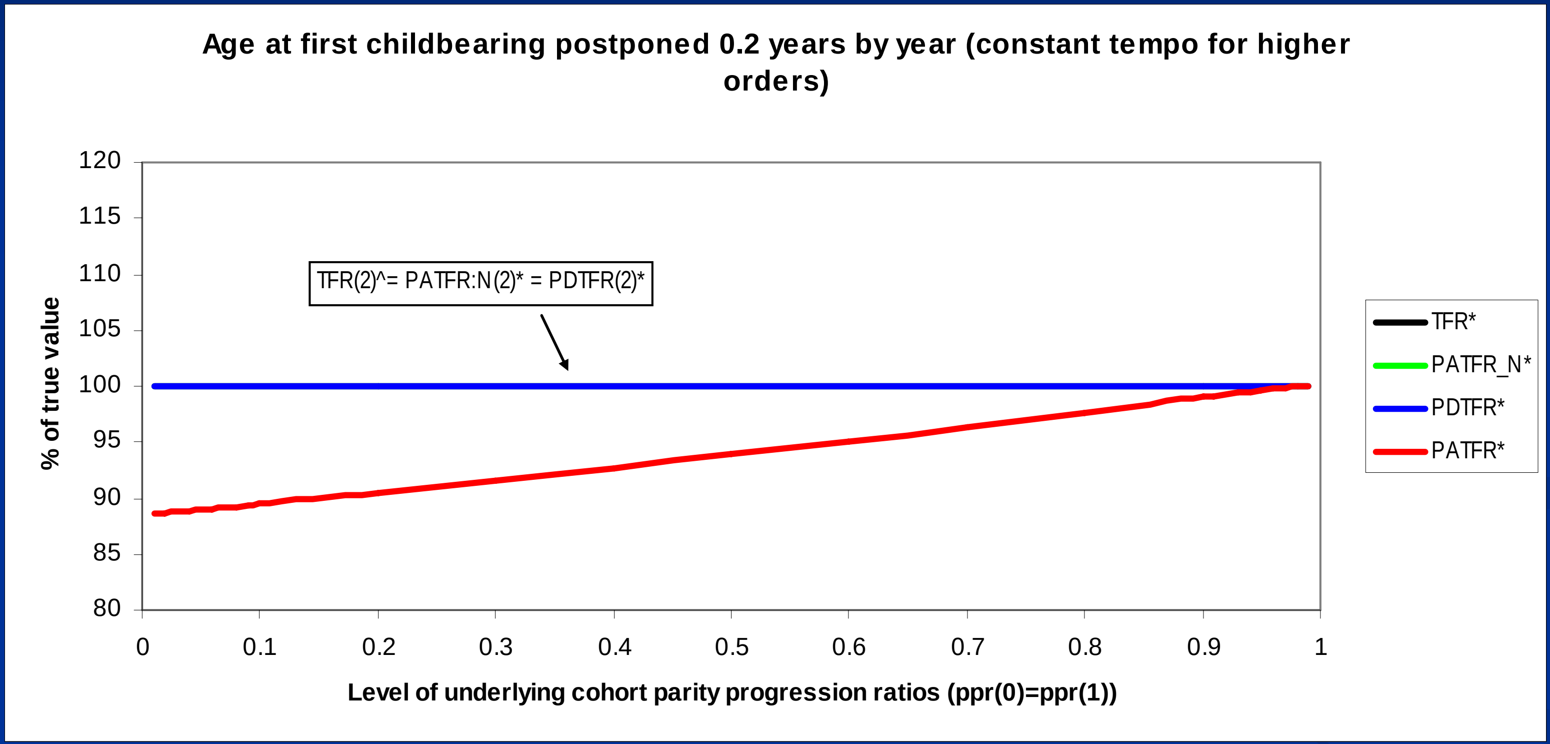
- PDTFR, the fertility index based *only* on duration since the last event, is always closer to the 'truth'.
- But all the Period Total Fertility indices *have to* be adjusted from tempo distortion if we want to measure correctly the level of fertility.
- We are going now to see what are the results of the adjustments, specially when there is no analytical relationship between cohort and period index (the case of PATFR at birth order 2 or more, for which we use Kohler and Ortega (2002) adjustment scheme).

Period Total Fertility indexes for 1st births, adjusted for tempo variation (in proportion of cohort level)



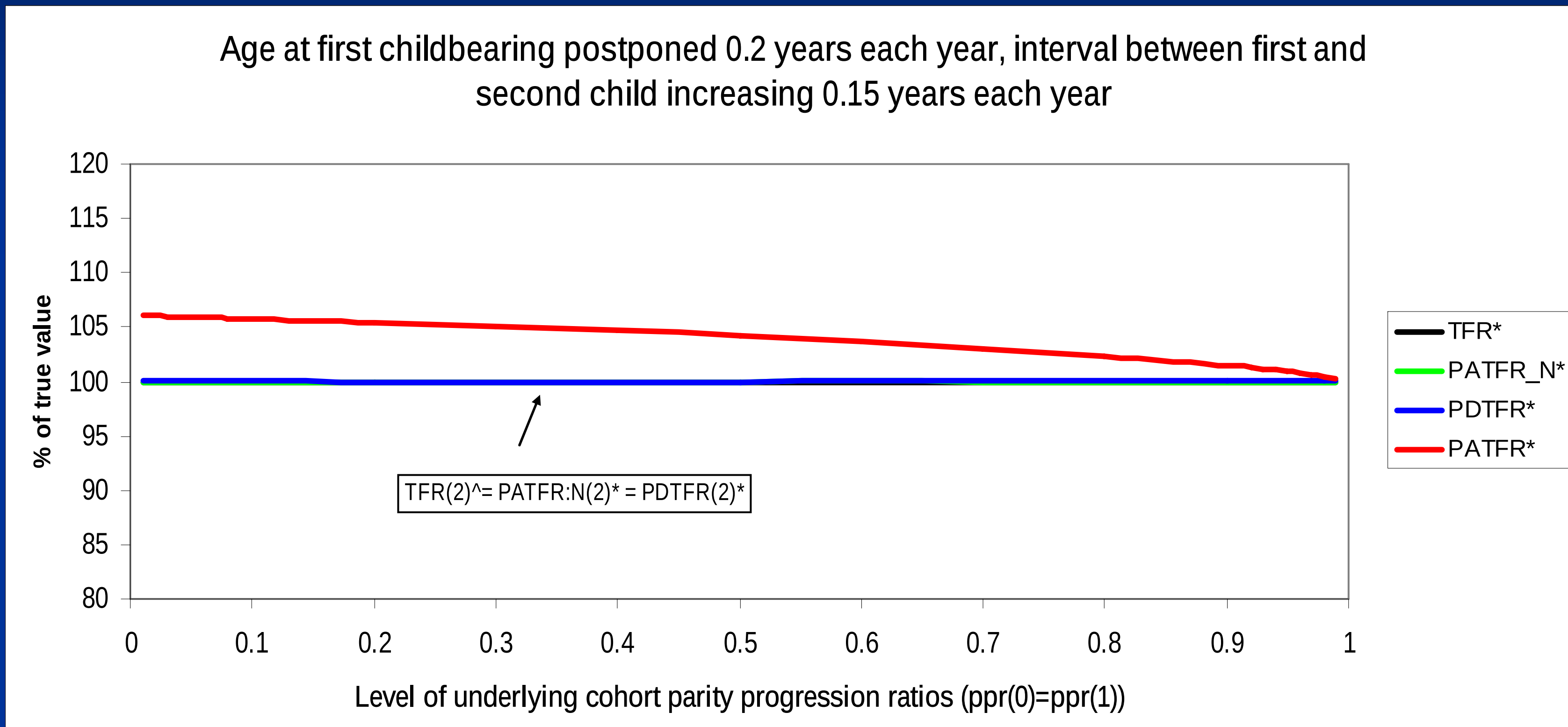
The adjustment formulas give perfect results (under the simplistic hypothesis of tempo variation)

Period Total Fertility indexes for 2nd births, adjusted for tempo changes (in proportion of cohort level) - I



The Kohler-Ortega correction is not enough to fully bring back the PATFR(2) to the *true* level.

Period Total Fertility indexes for 2nd births, adjusted for tempo changes (in proportion of cohort value) - II



When fertility is postponed both at the age and the duration of inter-births interval levels, the adjusted PATFR(2) overshoots its *true* level.

Conclusions

- The best (or closest to the *true* level) Total Fertility index is the one based on PATFR (age-parity) for the 1st birth order and on PDTFR (duration-parity) for the higher orders.
- Period fertility indexes are distorted by tempo variations, so they need to be adjusted in order to uncover the *true* level of fertility.
- We need to investigate better the sensitivity of the results in the case of non-linear tempo variations, like Zeng & Land (2001) have done for the Bongaarts-Feeney adjustment formula for the TFR.