A Cubic Model for Graduation of Life Tables

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Abstract: The graduation is a name for a class of techniques in the actuarial mathematics, which produce smooth estimates for probabilities of dying in life tables obtained from empirical data. The calculation of these probabilities for ages greater than 85 (e.g. at the end of the life table) is usually performed by the extrapolation of graduated values following Gompertz - Makeham law of mortality (the King - Hardy method). Nowadays, several authors have observed that this method overestimates the real probabilities. That is why other methods of extrapolation have been examined (Kannisto, Koschin and others). The purpose of the article is to present a graduation method, which in calculation process fits the logarithm of specific mortality rates by a cubic parabola. Moreover, alternative ways of ending the life table are proposed.

Key Words: life table, probability of dying, graduation methods, force of mortality, extrapolation, Gompertz - Makeham law

1. INTRODUCTION

Life tables represent a well-known basic tool of actuarial mathematics, which is efficiently applicable in other branches, as well. They contain the age specified probabilities of dying and express the subsequent dying out of the hypothetical population cohort. The key elements of the tables, the probabilities of dying, are estimated from the empirical statistical data. In accordance with the character of the used input data, several types of life tables can be distinguish (for details see e.g. Browers et al. 1986 or Keyfitz 1977). In this paper we shall due with the *complete* life tables, where the one-year age intervals are used.

The construction of life tables goes out from the set of empirical age specified rates of mortality $M_x = D_x / P_x$, where D_x is the observed number of dyed in the age interval $\langle x, x+1)$ during a given calendar year and P_x is the mean number of living persons aged $\langle x, x+1)$ in the midyear of the same calendar year, or more exactly, the number of person-years lived by the persons aged $\langle x, x+1)$ in this population, $x = 0, 1, ..., \omega$.

Using the above data, the empirical probabilities of dying are being calculated using one of the following two almost equivalent formulas

$$Q_x = \frac{M_x}{1+\frac{1}{2}M_x} \qquad \text{or} \qquad Q_x = 1 - \exp(-M_x) .$$

They can be considered as the starting (initial) estimations for the real probabilities, which are unknown. In the next step they are improved (smoothed) by some method of *graduation*. Usually, in the main part of tables (x = 4, ... 85) some kind of the moving average procedure is applied. For the old ages („at the end of the table“, $x > 85$), behind the modus of the number of dyed, the starting estimations are loaded with a great random error, and therefore extrapolation methods are being preferred. Until recently, only a small attention was paid to the construction of the end of the tables. Firstly, the actuarial praxis did not require it, because very old people have represented only a relatively small group of population and did not mean a significant risk for the insurance companies. Secondly, because of lack of the reliable data, the inaccuracies in computing methods were comparable with the random errors. Nowadays, as a consequence of the population ageing, the ratio of very old people has increased and they create an interesting group of potential clients for pension funds. Inaccuracies or overestimation of the probability of dying could significantly influence the economy of the funds. On the other hand, during the relatively long period of exact statistical evidence, a sufficiently great amount of reliably data was collected. Although neither today the quality of data is absolute, it enables to make more solid conclusions concerning the human mortality.

Until now, the statistical offices use various computation procedures, containing several simplifying assumptions. The tables are usually closed by an artificial assumption, that all persons will die before reaching certain age ω, i.e. by setting $l_x = 0$ for x large enough (e.g. $\omega = 101$ in ŠÚ SR 2002). Here, l_x is the number persons in hypothetical cohort at exact age x. For ages $x = 85, \ldots 100$ years the probabilities of dying are usually smoothed (*graduated*) by King-Hardy method (Pecka 1989, Mészáros 2000), which assumes the force of mortality in the form of modified exponential curve (Gompertz - Makeham law, see below). Recent investigations show that this assumption overestimates the real values, i.e. the probability of dying does not grow in very high ages so fast as the modified exponential supposes. For this reasons some attempts were made to replace the exponential curve by other, slower growing curve (Thather et al. 1998, Koschin 1999).

The aim of this paper is to present a new attempt with a new model for the force of mortality. The purpose is to obtain more realistic life table, which would be relatively stable in time and which would eliminate the mentioned overestimation of probabilities of dying in high ages. So they would be suitable for long-term prognoses, as well.

2. CHARACTERISATION OF USED INPUT DATA

Below we shall apply the new model on the recent data for Slovak Republic (SR). The data are collected by Statistical Office of the Slovak Republic (ŠÚ SR) and in general are considered as appropriately reliable. In the same time, on this data we shall illustrate its motivation and the possible problems connected with the standard approach.

By the calculation of initial estimations of probability of dying (empirical probabilities) Q_x a surprising fact occurs. These empirical probabilities do not growth nearly exponentially according to the Gompertz - Makeham model, but after a certain age (round 95 years) the do not grow at all. This is true not only for a single year (what could be caused by chance) but as a rule. For example, in 1999 for Slovak males Q_{98} decreased on the level as Q_{60}. Taking the confidence intervals for q_x , the hypothesis that q_x does not decrease is rejected.

The decrease of probability of dying in some age interval itself would be not a reason for doubt about the quality of data. This can be observed in many populations (e.g. in ages 0-10 or 21-24) and is biologically interpretable. The reason for distrust is other. For instance, the assumption $q_x < 0.4$ for $x > 98$ implies, that in SR would live much more centenarians than the statistical evidence shows. Therefore, the values $q_x < 0.4$ for $x > 100$ have to be regarded as unrealistic. On the other hand, $q_x = 0.6$ for $x > 100$ would lead to the conclusion, that in Slovakia there lives in average only 0.01 people aged 108 years, what contradicts the facts, as well (see ŠÚ SR 1998, ŠÚ SR 2002).

The causes of unreliability of input data for high ages may be various. First of all, in high ages the standard deviation of occurrence of events is relatively large, what causes great random errors (so called the „small numbers problem“). However, the differences between theoretical probabilities and their empirical estimations use to be larger than can be explained by randomness, both in males and in females. Thanks to the compulsory registration of deaths, their numbers seem to be relatively exact. Hence, the main source of inaccuracies should be searched in the numbers of living, which are evidently overestimated.

This phenomenon occurs not only in Slovakia. Similar discrepancies are reported also from Czech Republic (Koschin 1999) and other developed European countries (e.g. Thatcher et al. 1998).

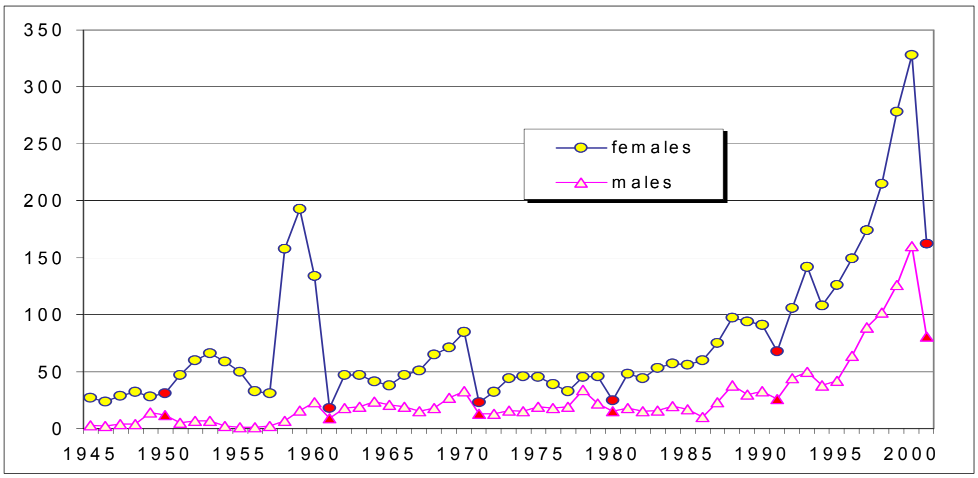

Fig. 1. Numbers of males and females aged 98 in SR, July 1st each year. Source: ŠÚ SR.

ŠÚ SR regularly publishes the midyear numbers of lives by age (e.g. ŠÚ SR 1999). The last figure (P_{100+}) is cumulated, and the distortion caused by the cumulative reflects also in figures for age 99. That is why we have examined the numbers of lives aged 98 in the second half of the 20th Century (Fig. 1.). The data from the year of Census are pointed up by a dark dot. As one can see, the figures in the year of Census are only a small portion of the figures before Census. This is probably a consequence of exact evidence of natural movement and inexact evidence of migration combined with inaccuracies of Census. The errors are cumulated in higher ages. For this reasons the standard empirical data are for $x > 95$ practically inapplicable. The only data of some value in this age group are those from Census. That is why the empirical basis for this paper creates the data from Census 2001.

3. SELECTION OF A MODEL

As usually, for $x \geq 0$ let us denote the point table variables (related to the exact age x): the number of living by l_x, the number of ever lived years for persons in the cohort by T_x, the life expectancy by e_x, and the force of mortality by μ_x, respectively, all at exact age.

Further, denote the following interval table variables: the number of died by d_x , the number of lived persons-years by L_x , the age specific mortality rate by $m_x = d_x / L_x$, and the probability of dying by $q_x = d_x / l_x$, respectively, all in the age-interval $\langle x, x+1)$. (For exact definitions and details see e.g. Keyfitz 1977, Browers et al. 1986, Cipra 1990). The above variables are tied by known relations

$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} ,$$

$$q_x = 1 - \exp\left(-\int_x^{x+1} \mu_t dt\right) \cong 1 - \exp(-\mu_{x+1/2}) \cong 1 - \exp(-m_x) .$$

As mentioned above, in the main part of tables (x = 4, ... 85) as the estimation of model probabilities of dying q_x the graduated empirical probabilities Q_x used to be taken. For high ages it is not very appropriate, because of both great random deviation in number of died and the mentioned inaccuracies in evidence of living. A more appropriate approach is the extrapolation of some parametric curve. Regarding the above remarks, the following types of fitting curves for μ_x or q_x should be taken into account:

a) (modified) exponential curve (Gompertz - Makeham law, method used by ŠÚ SR, too);

b) slower growing convex curve (Koschin 1999);

c) linear function (tangent line to the graph e.g. in $x = 85$); ????

d) growing concave curve (e.g. parabola with maximum behind age ω);

e) growing curve, first convex, then concave, with inflexion between $x = 85$ a $x = 98$ (in this paper);

f) constant (round $q_x = 0.35$ till 0.4, what corresponds to the force of mortality near to 0.5);

g) concave curve with local maximum behind $x = 95$;

h) S-curve (first concave, then convex) with possible local maximum round $x = 95$ and local minimum round $x = 101$.

Of course, some of these models might be difficult to interpret. Below, a series of models suitable for fitting the force of mortality $\mu_x = \mu(x)$ is given (for curves of type a, b, d, e).

1. Gompertz (1825)

$$\mu_x = B\,C^x \quad \Leftrightarrow \quad \ln \mu_x = \ln B + x \ln C = b + c\,x$$

2. Makeham (1860)

$$\mu_x = A + B\,C^x \quad \Leftrightarrow \quad \ln(\mu_x - A) = \ln B + x \ln C = b + c\,x$$

3. Weibull (1951)

$$\mu_x = B\,x^C \quad \Leftrightarrow \quad \ln \mu_x = \ln B + C \ln x$$

4. Kannisto (a specific logistic model 1992, similarly other authors, see Thather et al. 1998)

$$\mu_x = A + \frac{BC^x}{1+BC^x} \quad \Leftrightarrow \quad \text{logit}\,(\mu_x - A) = \ln B + x \ln C = b + c\,x$$

5. Koschin (1999), $\gamma > 0$, $x > x_0$

$$\mu_x = A + B \exp\{c\,(x_0 + \tfrac{1}{\gamma}\ \ln[\gamma(x-x_0)+1])\}$$

$$\Leftrightarrow \ \ln(\mu_x - A) = \ln B + (x_0 + \tfrac{1}{\gamma}\ \ln[\gamma(x-x_0)+1]\,)\ln C$$

6. Coale & Kisker (1990) (quadratic model, $d < 0$),

$$\mu_x = B\,C^x D^{x^2} \Leftrightarrow \ \ln \mu_x = \ln B + x \ln C + x^2 \ln D = b + c\,x + d\,x^2$$

7. cubic model ($e < 0$),

$$\mu_x = B\,C^x D^{x^2} E^{x^3}$$

$$\Leftrightarrow \ \ln \mu_x = \ln B + x \ln C + x^2 \ln D + x^3 \ln E = b + c\,x + d\,x^2 + e\,x^3$$

The type e) can be represented by model 4 or 7. As can be seen, the cubic model 7 is a natural extension of the generally used Gompertz type models. The transformed model (by logarithm or logit) enables to estimate the parameters by linear regression (model 1, 3, 6, 7). In particular, using the usual approximation $\mu_{x+1/2} \cong m_x$, from the cubic model (model 7)

$$\ln \mu_x = \ln B + x \ln C + x^2 \ln D + x^3 \ln E = b + c\,x + d\,x^2 + e\,x^3$$

we obtain the formula (4) for calculating probabilities of dying q_x as follows:

$$\ln \mu_{x+1/2} \cong \ln m_x = \beta + \gamma x + \delta x^2 + \varepsilon x^3$$

$$q_x = 1 - \exp(-m_x)$$

$$q_x = 1 - \exp(-\exp(\beta + \gamma x + \delta x^2 + \varepsilon x^3))$$

Coefficients β, γ, δ, ε are estimated by ordinary least square method from the model

$$\ln M_x = \beta + \gamma x + \delta x^2 + \varepsilon x^3 + \varepsilon_x$$

for a proper age interval (values of x).

Sensibility of the right end of the curve to random errors and inaccuracies is a common disadvantage of all models mentioned above. This can be significantly reduced when some proper condition on the right end is added. Below, an artificial value M_{100} fulfils this role. The unreliable observations M_x (e.g. for $x_0 \geq 95$) are substituted in regression model by one artificial observation $M_{100} = 0.5$.

The remarks in Section 2 concerning Slovak data imply that the observed numbers of living persons older than 100 years correspond to values of the probability of dying approximately $q_{100} = 0.4$ (or, almost equivalently, $\mu_{100} = 0.5$). This value may be common for other modern populations, too. Similar results have been achieved by searching the Czech data in several subsequent years (Koschin 1999). Kannisto with his research group collected and analysed in details the data from 13 developed European countries

(Kannisto 1998, Thather et al. 1998), their results confirm the above conclusions. For this reasons, q_{100} = 0.4 (or, alternatively, μ_{100} = 0.5) can be taken as a common condition in regression models, either absolutely (this point lies on the fitting curve) or as artificial input value Q_{100} = 0.4 or M_{100} = 0.5, respectively, having the same weight as other input data. This condition seems to be more realistic as the usual condition $L_\omega = 0$ (in fact, as follows from Meszáros 2000, ŠÚ SR takes $L_{101} = 0$). Thanks to it the described procedure is relatively universal, proper also for other contemporary populations, particularly for the small ones.

4. EXAMPLE - RESULTS FOR THE SLOVAK REPUBLIC DATA

We shall illustrate the described procedure on the Slovak Republic data for the 2001 (ŠÚ SR 2002). For the reasons discussed in Section 2, the data are taken for the year when the Census was performed, to ensure the greatest reliability. The calculation goes out of the empirical specific mortality rates M_x , x = 70, 71, ... 94 and the additional condition M_{100} = 0.5. This range of the input data was chosen in order to ensure the fluent switch between the main and the end parts of table, to catch the deceleration of the growth of mortality rates for very high ages, to minimize the distortion caused by unreliable data for high ages, and to enable the reasonable extrapolation for $x > 100$.

Using the cubic model (eq. (7)), the following relations were identified by OLS method:

Males: $\ln m_x = 22.8142 - 1.0578\, x + 0.0137\, x^2 - 5.44 \cdot 10^{-6} x^3$

Females: $\ln m_x = -0.5862 - 0.3447\, x + 0.006258\, x^2 - 2.82 \cdot 10^{-6} x^3$.

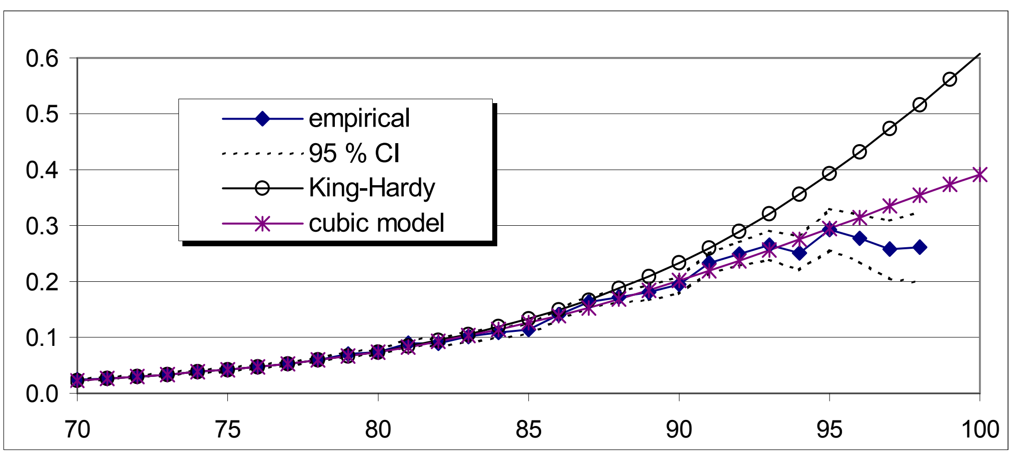

Fig. 2. Probability of dying in age x, females SR, 2001

The resulting graduated probabilities q_x (Tab. 1, Fig. 2) have been calculated then according the formula $q_x = 1 - \exp(-m_x)$. Fig. 2 displays the original empirical probabilities and related 95 % confidence intervals, as well as the Makeham-Gompetrz extrapolations obtained by classical King-Hardy method (ŠÚ SR) and the values graduated by the described conditioned cubic model for females. (For males SR the differences between models are smaller.) The both functions q_x (for males and females) are growing in x on reasonable age interval and reach the inflexion point and maximum as follows:

Males: inflexion point: $x = 94.76$ maximum: $q(106.83) = 0.4368$

Females: inflexion point: $x = 96.84$ maximum: $q(164.43) = 0.4248$.

The calculated probabilities for females have a more flat maximum, hence they are closer to the theoretical assumptions of the model than those for males.

The main parts of the tables were graduated by moving average method (two times centred polynomial moving average of third order and length 7).

As the last step, both the parts had to be joined. For ages x = 70, ... 94 years (the highest value of x, for which the moving averages could be calculated) the differences between q_x resulting from both methods were calculated. These differences oscillate round zero after some x_0, and the amplitudes tend to increase. Therefore, as the x_0 such a value x before oscillations was chosen that the absolute value of difference was minimal. Thus, the moving average was used last time for age x_0 = 78 years for males and x_0 = 77 years for females.

Table 1. Deaths (Dx), lives – midyear population (Px), empirical (ungraduated) estimation, graduation due the Gompertz – Makeham modified exponential model (King-Hardy) and graduation by conditioned cubic model, for age 70-94 years.

	Males, SR 2001					Females, SR 2001				
age x	Dx	Px	empirical	King-Hardy	cubic model	Dx	Px	empirical.	King-Hardy	cubic model
80	732	6793	0.102155	0.105059	0.105523	1014	13279	0.073518	0.074414	0.074394
81	742	5552	0.125100	0.113349	0.115565	1036	10951	0.090266	0.083650	0.083005
82	442	3613	0.115149	0.122278	0.126497	674	7134	0.090151	0.094017	0.092456
83	303	2036	0.138277	0.131889	0.135621	439	4116	0.101166	0.105637	0.102787
84	319	1868	0.156985	0.142222	0.145851	448	3893	0.108704	0.118642	0.114033
85	299	1955	0.141820	0.153322	0.158194	507	4187	0.114045	0.133168	0.126219
86	445	2331	0.173789	0.165231	0.169631	779	5130	0.140884	0.149360	0.139358
87	494	2295	0.193661	0.177994	0.182648	890	4987	0.163446	0.167365	0.153452
88	360	1822	0.179290	0.191655	0.195969	774	4103	0.171917	0.187334	0.168486
89	354	1386	0.225402	0.206255	0.210589	653	3285	0.180272	0.209411	0.184427
90	239	1017	0.209433	0.221836	0.226243	552	2558	0.194098	0.233734	0.201227
91	223	794	0.244863	0.238436	0.242446	541	2039	0.233044	0.260427	0.218815
92	186	577	0.275560	0.256089	0.259080	421	1470	0.249034	0.289589	0.237101
93	106	381	0.242866	0.274826	0.276007	314	1021	0.264748	0.321287	0.255974
94	95	263	0.303172	0.294669	0.293068	210	725	0.251478	0.355542	0.275306
95	60	180	0.283469	0.315636	0.310087	180	519	0.293068	0.392320	0.294948
96	30	134	0.200589	0.337733	0.326871	125	385	0.277238	0.431513	0.314738
97	25	94	0.233528	0.360959	0.343219	74	249	0.257096	0.472930	0.334499
98	18	81	0.199263	0.385296	0.358916	49	162	0.261009	0.516281	0.354048
99	14	99	0.131870	0.410716	0.373749	39	190	0.185567	0.561165	0.373194
100+	9	85	0.100470	0.437172	0.387499	45	164	0.239965	0.607070	0.391743

5. LAST ROW OF THE TABLE

When substitute the obtained probabilities of dying into the life table, the rest of the table can be completed (see e.g. ŠÚ SR 2002). For comparison, doing this, we get the life expectancy e_0 greater in 0.14 year for females, but for males less in 0.04, than when using the classical Gompertz-Makeham model (King-Hardy method). The life expectancy in age $x = 80$ is then in 0.32 year greater, but for males in 0.09 lower. The life expectancy in age $x = 90$ is for females even in 0.58 year greater, but for males in 0.01 less lower. Then the last row (x = 100+) of the life tables calculated by the ŠÚ SR methodology would be as it is shown in Tab. 2.

Table 2. Last rows in life tables with probabilities of dying graduated by conditional cubic model and closed according to the methodology of ŠÚ SR, SR 2001.

x	q_x	p_x	l_x	d_x	L_x	T_x	e_x
Males:							
100+	0.387499	0.612501	134	134	108	108.4	0.81
Females:							
100+	0.391743	0.608257	455	455	366	366.1	0.80

In fact, methodology of the Statistical Office of Slovak Republic (ŠÚ SR) supposes that $\omega = 101$ is a limit age, e.i. nobody older survives it ($l(x) = 0$ for $x > \omega$). The former Czechoslovak Federal Statistical Office (FSU, see Pecka 1989) calculated the life tables in the same way with $\omega = 103$. This approach may be written in the form

X	q_x	p_x	l_x	d_x	L_x	T_x	e_x
ω-1	$q_{\omega-1}$	$1- q_{\omega-1}$	$l_{\omega-1}$	$l_{\omega-1}\, q_{\omega-1}$	$l_{\omega-1}\,(1-q_{\omega-1}/2)$	$l_{\omega-1}\,(1-q_{\omega-1}/2)$	$1-q_{\omega-1}/2$
ω	1	0	$l_{\omega-1}- d_{\omega-1}$	$l_{\omega-1}- d_{\omega-1}$	0	0	0

or, using the cumulative form,

$(\omega$-1)+	$q_{\omega-1}$	$1- q_{\omega-1}$	$l_{\omega-1}$	$l_{\omega-1}$	$l_{\omega-1}\,(1-q_{\omega-1}/2)$	$l_{\omega-1}\,(1-q_{\omega-1}/2)$	$1-q_{\omega-1}/2$

The recent alternative approach used by some authors (see e.g. in Cipra 1990) understands ω as the begin of the last, unlimited age interval $\langle \omega ; \infty)$. We shall show, how then the last row looks out when in addition to it the constant force of mortality μ on this interval is supposed. Supposing this, the number of living l_x decreases exponentially, therefore the number of person-years L_x lived in this interval is equal

$$L_x = \int_x^{x+1} l_t dt = \int_0^1 l_x e^{-\mu t} dt = l_x \ (e^0 - e^{-\mu})/\mu = (1- e^{-\mu}) \ l_x /\mu$$

and similarly

$${}_\infty T_x = {}_\infty L_x = l_x / \mu \, .$$

This means among others, that, in the relation $L_x = l_{x+1} + a_x\, d_x$, the average number of years lived in the interval $\langle x; x+1)$ by persons who died in this interval is

$$a_x = \frac{1}{\mu} - \frac{1}{e^{\mu} - 1} \ .$$

Under this conditions the $e_x = 1/\mu$ remains constant (e.g. the person aged x will survive in average still $1/\mu$ years, independently of his/her age). Then the row ω of the table looks out as follows (here l_ω is obtained from the previous row)

X	q_x	p_x	l_x	d_x	L_x	T_x	e_x
ω	$1\text{-}e^{-\mu}$	$e^{-\mu}$	l_ω	$(1\text{-}e^{-\mu})\, l_\omega$	$(1\text{-}\, e^{-\mu})\, l_x /\mu$	l_x /μ	$1/\mu$

or, when considered as a cumulative row (in this case d_x, L_x, T_x are related to the whole interval $\langle \omega; \infty)$):

x	q_x	p_x	l_x	d_x	L_x	T_x	e_x
ω+	$1\text{-}e^{-\mu}$	$e^{-\mu}$	l_ω	l_ω	l_x /μ	l_x /μ	$1/\mu$

For the population of Slovak Republic, the additional assumption $\mu_x = \mu = 0{,}5$ for $x > \omega = 101$ gives $a_x = 0{,}4585$. Then, when l_{101} is calculated in the usual way, the last (cumulative) row has a form shown in Tab. 3.

Table 3. Last rows in life tables with probabilities of dying graduated by conditional cubic model and closed by unlimited interval $\langle \omega; \infty)$ with constant force of mortality $\mu = 0.5$ on this interval.

x	q_x	p_x	l_x	d_x	L_x	T_x	e_x
Males:							
101+	0.393469	0.606531	82	82	165	165	2.00
Females:							
101+	0.393469	0.606531	277	277	554	554	2.00

It is interesting, that the same fixed value $e_{100} = 2.0$ is stated in the abbreviated life tables published by ŠÚ SR until 1998 (and the FSU until 1992), and calculated by former methodology. However, the reasons for it were others.

Finally, let us note, that the life table shows the age structure of the stationary population. Thus, while according to the life table using Gompertz-Makeham model (ŠÚ SR), the females aged 80 and more years will create 4.5 % of stationary population, according the method presented here it will be even 5.8 % (for males the difference is less).

6. CONCLUSION

Conditioned cubic model presented in this paper (the cubic model for logarithm of the force of mortality, with an additional condition on M_{100}) enables simple and realistic estimation of probabilities of dying in high ages. The suggested method seems to be relatively universal and suitable for small populations, as well.

The crucial point of the method is the way of closing the table. There are good reasons for the simplifying assumption about the constant force of mortality in the age over 100 years, because it assures a more realistic close of the table than the often used, but also artificial assumption $L_{101} = 0$. The choice $\mu = 0.5$ for $x > 100$ considered above could be more questionable. As confirm the observations of Kannisto (1998), calculations of mortality for very old ages ($x > 100$) would require more precise model.

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