Migration as a Factor of Population Reproduction

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Abstract

Completed Net Migration is introduced as an expected number of net migrants to join the original birth cohort through its life span. In the context of population reproduction, Migratory Fertility is introduced as a measure of migrants' addition to the reproduction of population birth cohorts. Robust estimating procedures are suggested which reflect the differential fertility of migrants and also significant links between migrations at different ages. Combined Fertility, which is approximated by the sum of conventional Total Fertility and of Migratory Fertility, characterises the reproduction regimen of a population with the migratory impact on reproduction explicitly taken into account. The concept of stable equivalent population is also adjusted to include the migratory impact on population growth and structure, which provides a sound basis for studying the prospects of population ageing.

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1. INTRODUCTION

Classical demographic indicators were developed in an era of high fertility and mortality and of low migration. Hence, they put the stress on fertility and mortality and considered migration as an additive to the natural reproduction process. Widely-used measures of reproduction—the *Total Fertility* (*TF*) and *Net Reproduction* (*NR*)[1]—convey no information on migration at all. The first indicator tells us how many births a baby might produce during her life under fixed age-specific fertility rates and no mortality. The second indicator adds the effects of mortality and shows how many births of its own sex a baby might produce during its life span under fixed age-specific fertility rates and survival regimen. Both indicators show how the cohort replaces itself in the next generation. Population reproduction has changed dramatically since the 19th and the beginning of 20th century, when classic formal demography was flourishing. These days low mortality is common while migration breaks historical records in many countries (Coleman 2006). For modern populations with low mortality the link between *TF* and *NR* has become so simple that one might doubt whether it is still necessary to keep both indicators in the toolkit. Indeed, without any in-depth study of fertility rates and of life tables, one may state that *TF* of around 2.1 implies simple replacement of cohorts (i.e., *NR*=1) for a low-mortality population. On the background of this picture of fertility and mortality, possibly over-detailed for modern low-mortality populations, classical demography says nothing about migration as a factor of population replacement.

Several approaches were proposed in the literature to clarify migrational impact on population reproduction. Keely (1974) pointed to many problems and potential biases in interpreting the immigration role in population growth based on crude data. Hyrenius (1951) undertook a more

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[1] These indicators are widely known as Total Fertility *Rate* and Net Reproduction *Rate*. However, they are not rates in 'pure' sense, and we omit the term 'rate' unlike in many other works.

thorough approach, adding age-specific migration rates into the context of replacement in the same traditional manner as mortality rates are used to derive survival rates and the net replacement rate. He introduced a concept called the *social replacement rate*, which measured the number of children born to a girl through her life span, taking into account her survival probabilities and also probabilities of migrating (treating immigration in the same way as emigration and mortality). This concept is quite close to some of the indices proposed here. However, it ignores some important internal regularities in migration, fertility differentials and also may not be robust enough to be used in the applied study of migration. Keyfitz (1977) studied the possible usage of constant-rate emigration at a given age as a means of population growth control. Espenshade (1984, 1986) also considered constant-rate migrations and added them to mortality in the same way as Hyrenius. Rogers (1975) used a similar approach to modelling internal inter-regional migrations. Sivamurthy (1982) introduced another model considering the constant crude rate of net migration and fixing the age profile of net migrants. He studied consequences for population growth, stable age structure, and also for ergodicity. Reviewing the aforementioned work, Mitra (1984) proposed a simpler, yet nearly equivalent formulation, assuming overall migration to be proportional to the number of births instead of the population size. Other researchers treated migration as being constant in absolute terms or varying, but independent of the population age structure (Coale 1972; Pollard 1973; Espenshade, et. al. 1982; Mitra 1983, 1990; Cerone 1987; Arthur and Espenshade 1988; Blanchet 1989; Schmertmann 1992), considering the consequences of migration on stable population with a specific interest in population ageing. Bouvier, Poston and Zhai (1997) studied the effects of zero net migration on population person years and on births, and demonstrated that a simplistic approach to the issue may result in erroneous conclusions. Direct and indirect effects of migration were also studied in the context of urbanisation (e.g., Martine 1975). More recently, a UN approach to estimate ‘replacement migration’ necessary either to support the constant population size or to prevent from decline of the labour force has pulled attention to the role of migration in population reproduction (United Nations Population Division 2000). Such an approach based on scenario projections, however, lacks clarity in understanding the role of migration in population reproduction and also is sensitive to scenario settings. Hence, there is still a need for demographic indicators which might quantify the role of migration in population reproduction.

Given the increased role of migration and the need for better demographic analysis for migration policy making, this paper addresses the need to develop new indicators of population reproduction which put migration into the same cohort reproduction context as was classically done for fertility and mortality.

2. MIGRATION IN THE LIFE COURSE OF POPULATION COHORTS

As in the case of traditional fertility and reproduction measures, which are based on a hypothetical scenario with age-specific fertility rates constant at the observed levels, we also develop our indicators assuming age-specific net migration rates to be constant[2]. Within this framework one may estimate, for example, the number of net migrants expected to join the indigenous birth cohort during the entire life span of the cohort, calculated per one birth in the cohort and assuming fixed age-specific net migration rates. We denote this number by *CNM* which stands for *Completed Net Migration.* Neglecting mortality—as is traditionally done for obtaining *TF*—one may

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[2] Similar to the case of fertility indicators, this assumption has both advantages and drawbacks in studying actual migration. Indicators developed from it provide clearly interpretable estimations of the role migration might play in population dynamics under continuation of the observed rates to future. They help to quantify the importance of observed migration rates for population dynamics. However, they do not indicate causes behind observed rates and also factors, which may affect migration in future. Note, however, that in the section concerning the role of migration for population reproduction, we will improve the migration model by taking explicitly into account the links between migrations at different ages (e.g., the number of migrating children depends on the migration of parents, rather than on the number of children in the receiving population).

derive *Completed Gross Migration (CGM).* In order to clarify the concept, let us consider the following illustrative example.

Table 1 Illustrative example of net migration adding to a cohort of 1000 indigenous births. Annual migration equals 10 at ages 0-39, there is no net migration at ages 40+, and mortality is ignored.

Age	**Original birth cohort** [a]	**Net migrants joined the population at ages *x* to *x*+5**	**Cumulated net migrants by age *x***	**Population at exact age *x***	**Population at exact age *x*+5**	**Total person-years at ages *x* to *x*+5**	**Age-specific net migration rate, per 1000**
(*x*)	**(1)**	**(2)**	**(3)**	**(4)= (1)+(3)**	**(5)= (4)+(2)**	**(6)= 5*((4)+(5))/2**	**(7)= 1000*(2)/(6)**
0	1000	50	0	1000	1050	5125	9.76
5	1000	50	50	1050	1100	5375	9.30
10	1000	50	100	1100	1150	5625	8.89
15	1000	50	150	1150	1200	5875	8.51
20	1000	50	200	1200	1250	6125	8.16
25	1000	50	250	1250	1300	6375	7.84
30	1000	50	300	1300	1350	6625	7.55
35	1000	50	350	1350	1400	6875	7.27
40	1000	0	400	1400	1400	7000	0.00
45	1000	0	400	1400	1400	7000	0.00
50	1000	0	400	1400	1400	7000	0.00
55	1000	0	400	1400	1400	7000	0.00
60	1000	0	400	1400	1400	7000	0.00
65	1000	0	400	1400	1400	7000	0.00
70	1000	0	400	1400	1400	7000	0.00
75	1000	0	400	1400	1400	7000	0.00
80	1000	0	400	1400	1400	7000	0.00
85	1000	0	400	1400	1400	7000	0.00
90	1000	0	400	1400	1400	7000	0.00
95	1000	0	400	1400	1400	7000	0.00
100	1000	-	400	1400	-	-	-

[a] Only the original size of the birth cohort is shown. The actual number of the indigenous population may be subject to changes due to migration (mortality is ignored).

We consider a birth cohort of 1000 births, which is joined by 10 net migrants each year up to the age of 40, with nil net migration at ages above 40. Assuming no mortality[3], we may derive a 'net migration table' (Table 1) which shows how net migrants are adding to the cohort. In this example 1000 indigenous births are joined by 400 net migrants by the end of the life span of the cohort. This yields $CNM = 0.4$, i.e., 0.4 net migrants join a person born in the population, during the entire life span. Note that the phrase "join a person born in the population" is figurative, as indigenous persons themselves may emigrate. One could deepen the analysis, by distinguishing between net migration of indigenous inhabitants and of others[4]. We do not attempt this task, however, because the indicators developed from net migration rates are interesting *per se* and also relevant to the prospective importance of migration in population dynamics. Also note that the age-specific net migration rates presented in the last column of Table 1 do not depend on the composition of net migration flows according to indigenous or immigrant origin.

No matter how simple and transparent the additions by net migration might seem from the illustrative example above, it is quite complicated to estimate it from data on real population. Indeed, for real populations one may seldom have initial data for real cohorts in the form of Table 1. In addition, just as in the case of fertility indicators, these might be hypothetical calendar 'cohorts', which are of interest in the study of the most recent trends in net migration. Such studies are based on rates observed at a specific calendar year, which are assigned to a hypothetical birth cohort that might follow the same rates in its life span. Therefore, these are age-specific net

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[3] Here we neglect mortality for the sake of simplicity as net migration is observed at young ages only in the example presented. Further down mortality will be included into analysis.

[4] This may easily be done by decomposing the indicators presented in this work into a sum of indicators computed for the native-born and for non-native populations. See the Appendix for illustrations.

migration rates, which are usually available for studying the completed net migration. It might seem attractive to obtain the completed net migration as the sum of age-specific net migration rates—just like the *TF* is obtained as a mere sum of age-specific fertility rates. In the case of migration, however, such straightforward computation may lead to serious biases. In the illustrative case presented, for example, summing up age-specific net migration rates (and multiplying the sum by the duration of the basic age interval, i.e., by five) yields 336 net migrants, which clearly underestimates the actual number of net migrants joining the cohort (400). The source of this underestimation lies in the fact that age-specific net migration rates are calculated as net migrants per mid-year population in the respective age group. As the population consists of both the indigenous cohort population and of those who immigrated in previous years, the net migration rates will be deflated compared to the ratios of net migrants to the '*indigenous'* population only. For a population experiencing net out-migration, by contrast, age-specific net migration rates will tend to overestimate the role of migration in the life span of the cohort. On the other hand, small numbers of survivors to oldest-old ages will result in migration rates significantly higher in absolute value compared to migration according the original size of the birth cohort. Therefore, age-specific net migration rates should be corrected before being summed in order to get un-biased *Completed Net Migration*. These corrections depend on how the proportion of indigenous births in the total population changes throughout the life span of the cohort due to migration and survival.

We derive age-specific proportions of indigenous births in the total population assuming net migration and deaths to be evenly distributed within the tabulated age groups and assuming no mortality for migrants in the age interval during which they migrate. Hence, age-specific net migration rates are given by the ratios:

$$ {}_{\Delta}m_x = \frac{{}_{\Delta}M_x}{\Delta\cdot\left(\frac{1}{2}\left(1+{}_{\Delta}p_x\right)N_x + \frac{1}{2}\,{}_{\Delta}M_x\right)}, \qquad (1) $$

here Δ is the length of elementary age interval considered, ${}_{\Delta}p_x$ is the probability of survival from exact age x to exact age $x+\Delta$, N_x is the population at exact age x, and ${}_{\Delta}M_x$ is the net migration at ages x to $x+\Delta$. Net migration rates (1) show net migration relative to the person-years lived in the population at respective ages. One may also derive other rates, showing net migration in some age interval in respect to the population at the beginning of the age interval:

$$ {}_{\Delta}\pi_x = \frac{{}_{\Delta}M_x}{\Delta\cdot N_x} = \frac{1}{2}\left(1+{}_{\Delta}p_x\right)\frac{{}_{\Delta}m_x}{1-\frac{1}{2}\Delta\cdot{}_{\Delta}m_x}. \qquad (2) $$

Using these rates, in turn, it is possible to derive explicitly the proportion of the original births in the total population of the cohort. Let ${v^{ab}}_x$ be the proportion of the original births (*B*) in the population at exact age *x*:

$$ {v^{ab}}_x = \frac{B}{N_x}. \qquad (3) $$

As this relation applies to all ages, we may derive:

$$ {v^{ab}}_x = \frac{{v^{ab}}_{x-\Delta}\cdot N_{x-\Delta}}{N_x} = {v^{ab}}_{x-\Delta}\,\frac{N_{x-\Delta}}{{}_{\Delta}p_{x-\Delta}\cdot N_{x-\Delta} + {}_{\Delta}M_{x-\Delta}} = $$

$$ = {v^{ab}}_{x-\Delta}\,\frac{1}{{}_{\Delta}p_{x-\Delta} + \Delta\,{}_{\Delta}\pi_{x-\Delta}} = {v^{ab}}_{x-\Delta}\,\frac{1-\frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-\Delta}}{{}_{\Delta}p_{x-\Delta} + \frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-\Delta}}. \qquad (4) $$

Following this recurrent relation, we may establish the following explicit formula relating the share of indigenous births in the population to net migration rates:

$$ {v^{ab}}_x = {v^{ab}}_{x-\Delta}\,\frac{1-\frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-\Delta}}{{}_{\Delta}p_{x-\Delta} + \frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-\Delta}} $$

$$ = {v^{ab}}_{x-2\Delta}\,\frac{1-\frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-2\Delta}}{{}_{\Delta}p_{x-2\Delta} + \frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-2\Delta}}\cdot\frac{1-\frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-\Delta}}{{}_{\Delta}p_{x-\Delta} + \frac{1}{2}\Delta\cdot{}_{\Delta}m_{x-\Delta}} = \ldots $$

$$
\begin{aligned}
&= \nu^{ab}{}_0 \frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_0}{{}_\Delta p_0+\frac{1}{2}\Delta\cdot{}_\Delta m_0}\cdot\frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_\Delta}{{}_\Delta p_\Delta+\frac{1}{2}\Delta\cdot{}_\Delta m_\Delta}\cdot \\
&\ldots\cdot\frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_{x-2\Delta}}{{}_\Delta p_{x-2\Delta}+\frac{1}{2}\Delta\cdot{}_\Delta m_{x-2\Delta}}\cdot\frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_{x-\Delta}}{{}_\Delta p_{x-\Delta}+\frac{1}{2}\Delta\cdot{}_\Delta m_{x-\Delta}} \\
&= \frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_0}{{}_\Delta p_0+\frac{1}{2}\Delta\cdot{}_\Delta m_0}\cdot\frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_\Delta}{{}_\Delta p_\Delta+\frac{1}{2}\Delta\cdot{}_\Delta m_\Delta}\cdot\ldots\cdot\frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_{x-2\Delta}}{{}_\Delta p_{x-2\Delta}+\frac{1}{2}\Delta\cdot{}_\Delta m_{x-2\Delta}}\cdot \\
&\cdot\frac{1-\frac{1}{2}\Delta\cdot{}_\Delta m_{x-\Delta}}{{}_\Delta p_{x-\Delta}+\frac{1}{2}\Delta\cdot{}_\Delta m_{x-\Delta}} \qquad (5)
\end{aligned}
$$

as there are no migrants at the moment of birth of the cohort, i.e., $\nu^{ab}{}_0 = 1$.

Now we may derive age-specific net migration rates with respect to the original size of the birth cohort:

$$
{}_\Delta\delta_x = \frac{{}_\Delta M_x}{\Delta\cdot B} = \frac{{}_\Delta M_x}{\Delta\cdot\nu^{ab}{}_x\cdot N_x} = \frac{1}{\nu^{ab}{}_x}{}_\Delta\pi_x = \frac{\frac{1}{2}\left(1+{}_\Delta p_x\right)}{\nu^{ab}{}_x\left(1-\frac{1}{2}\Delta\cdot{}_\Delta m_x\right)}\cdot{}_\Delta m_x . \qquad (6)
$$

This relation gives explicitly the corrections that are to be applied to the age-specific net migration rates before summing them in order to get the *Completed Net Migration*. Summing up all age-specific ‘cohort joining’ rates (6), we may obtain the *CNM*, i.e., the expected number of net migrants to join the indigenous birth cohort during the entire life span of the cohort, calculated per one birth in the cohort and assuming fixed age-specific migration rates[5]:

$$
\begin{aligned}
CNM &= \Delta\cdot\sum_x {}_\Delta\delta_x = \Delta\cdot\sum_x\frac{1}{\nu^{ab}{}_x}\cdot{}_\Delta\pi_x \\
&= \Delta\cdot\sum_x\frac{\frac{1}{2}\left(1+{}_\Delta p_x\right)}{\nu^{ab}{}_x\left(1-\frac{1}{2}\Delta\cdot{}_\Delta m_x\right)}\cdot{}_\Delta m_x . \qquad (7)
\end{aligned}
$$

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[5] For the sake of simplicity, we do not explicitly consider the sex in the relations presented, although they may be written for both sexes separately, for both sexes combined, and also for two-sex populations using age/sex-specific net migration rates.

Table 2 Calculations of the *Completed Net Migration* for an illustrative net migration scenario (see Table 1 for description of the scenario).

Age	**Age-specific net migration rate, per 1000 population**	**Proportion of size at birth of the indigenous cohort in population at exact age *x***		**Adjusted age-specific net migration rate, per 1000 births in the indigenous cohort**	
		exact relation (7)	approximate (10)	exact relation (7)	approximate (10)
(*x*)	**(1)**	**(2)**	**(3)**	**(4)= (1)/((2)*(1-0.25*(1)/1000))**	**(5)= (1)/((3)*(1-0.25*(1)/1000))**
0	9.76	100.00%	100.00%	10.0000	10.0000
5	9.30	95.24%	95.24%	10.0000	9.9999
10	8.89	90.91%	90.91%	10.0000	9.9998
15	8.51	86.96%	86.96%	10.0000	9.9997
20	8.16	83.33%	83.34%	10.0000	9.9997
25	7.84	80.00%	80.00%	10.0000	9.9996
30	7.55	76.92%	76.93%	10.0000	9.9996
35	7.27	74.07%	74.08%	10.0000	9.9995
40	0.00	71.43%	71.43%	0.0000	0.0000
45	0.00	71.43%	71.43%	0.0000	0.0000
50	0.00	71.43%	71.43%	0.0000	0.0000
55	0.00	71.43%	71.43%	0.0000	0.0000
60	0.00	71.43%	71.43%	0.0000	0.0000
65	0.00	71.43%	71.43%	0.0000	0.0000
70	0.00	71.43%	71.43%	0.0000	0.0000
75	0.00	71.43%	71.43%	0.0000	0.0000
80	0.00	71.43%	71.43%	0.0000	0.0000
85	0.00	71.43%	71.43%	0.0000	0.0000
90	0.00	71.43%	71.43%	0.0000	0.0000
95	0.00	71.43%	71.43%	0.0000	0.0000
100	-	71.43%	71.43%	-	-
Total:	**336.42**			**400.00**	**399.99**

For the usual case of relatively small age-specific net migration rates and of low mortality at ages with considerable migration, the relations presented above may be substituted by simpler approximations:

$$v^{ab}{}_x \approx \frac{1}{{}_x p_0} e^{-\Delta \sum_{y=0}^{x-\Delta} {}_\Delta m_y}, \tag{8}$$

$${}_\Delta \delta_x \approx {}_x p_0 \cdot e^{\Delta \sum_{y=0}^{x-\Delta} {}_\Delta m_y} \cdot {}_\Delta \pi_x = \frac{\frac{1}{2}(1 + {}_\Delta p_x)\,{}_x p_0 \cdot e^{\Delta \sum_{y=0}^{x-\Delta} {}_\Delta m_y}}{1 - \frac{1}{2}\Delta \cdot {}_\Delta m_x} \cdot {}_\Delta m_x, \tag{9}$$

$$CNM \approx \Delta \cdot \sum_x \frac{\frac{1}{2}(1 + {}_\Delta p_x)\,{}_x p_0 \cdot e^{\Delta \sum_{y=0}^{x-\Delta} {}_\Delta m_y}}{1 - \frac{1}{2}\Delta \cdot {}_\Delta m_x} \cdot {}_\Delta m_x\ . \tag{10}$$

Table 2 demonstrates the relations obtained for the illustrative example presented above (note that mortality is ignored in the example, and all the formulas presented may be simplified). As one may note, the correction procedure does allow converting conventional age-specific net migration rates into rates per ‘indigenous births’. Also notable, exponential approximation is quite effective, resulting in discrepancies in the fifth digit only. Summing age-specific net migration rates—on the contrary—might result in a significantly biased estimate of *CNM*. These conclusions are also supported from calculations based on real data.

Tables 3 and 4 present calculations for the female population of Vienna in 2004, without and with mortality. Due to the small population numbers at oldest ages, it is necessary to take mortality into account, as can be seen comparing the results in the tables. Alternatively, one might simply ignore net migration at ages above some ‘maximal migration’ age. Our results suggest that the age of 80-85 may be taken as such a maximum age. Vienna has had a huge *CNM* of 2085, which exceeds the original size of the birth cohort by more than 100%.

Table 3 Calculations of the *Completed Net Migration* for an illustrative net migration scenario (net migration rates are derived from data on Vienna, females, 2004; no mortality).

Age	Survivors of the original birth cohort	Age-specific net migration rate, per 1000 mid-age population	Age-specific net migration rate, per 1000 population at exact age *x*	Proportion of size at birth of the indigenous cohort in population at exact age *x*		Adjusted age-specific net migration rate, per 1000 births in the indigenous cohort	
				exact	approximate	exact	approximate
(*x*)	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)=(3)/(4)**	**(7)=(3)/(5)**
0	1000	10.9	11.2	100.0%	100.0%	11.2	11.2
5	1000	9.5	9.8	94.7%	94.7%	10.3	10.3
10	1000	10.7	11.0	90.3%	90.3%	12.2	12.2
15	1000	60.5	71.3	85.6%	85.6%	83.3	83.3
20	1000	90.3	116.6	63.1%	63.2%	184.9	184.4
25	1000	49.1	56.0	39.9%	40.3%	140.6	139.1
30	1000	10.6	10.9	31.1%	31.5%	35.0	34.6
35	1000	-1.6	-1.6	29.5%	29.9%	-5.4	-5.3
40	1000	2.5	2.5	29.8%	30.1%	8.6	8.5
45	1000	2.1	2.1	29.4%	29.7%	7.2	7.2
50	1000	1.6	1.6	29.1%	29.4%	5.4	5.3
55	1000	-3.2	-3.2	28.9%	29.2%	-11.1	-11.0
60	1000	-3.8	-3.7	29.3%	29.7%	-12.7	-12.6
65	1000	-1.1	-1.1	29.9%	30.2%	-3.6	-3.6
70	1000	-2.0	-2.0	30.0%	30.4%	-6.7	-6.6
75	1000	-4.0	-4.0	30.3%	30.7%	-13.0	-12.9
80	1000	-6.5	-6.4	31.0%	31.3%	-20.7	-20.5
85	1000	-12.1	-11.8	32.0%	32.4%	-36.8	-36.4
90	1000	-12.1	-11.8	34.0%	34.4%	-34.7	-34.3
95	1000	-12.1	-11.8	36.1%	36.5%	-32.6	-32.3
100	1000			38.4%	38.8%		
Total:		**946**	**1178**			**1606**	**1603**

Source: Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.

Table 4 Calculations of the *Completed Net Migration* for an illustrative net migration scenario (net migration rates are derived from data on Vienna, females, 2004).

Age	**Survivors of the original birth cohort**	**Age-specific net migration rate, per 1000 mid-age population**	**Age-specific net migration rate, per 1000 population at exact age *x***	**Proportion of size at birth of the indigenous cohort in population at exact age *x***		**Adjusted age-specific net migration rate, per 1000 births in the indigenous cohort**	
				exact	approximate	exact	approximate
(*x*)	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6) =(3)/(4)**	**(7) =(3)/(5)**
0	1000	10.9	11.2	100.0%	100.0%	11.2	11.2
5	996	9.5	9.8	95.1%	95.1%	10.3	10.3
10	996	10.7	11.0	90.7%	90.7%	12.2	12.2
15	995	60.5	71.2	86.0%	86.0%	82.8	82.8
20	994	90.3	116.5	63.5%	63.7%	183.6	183.1
25	992	49.1	56.0	40.1%	40.6%	139.4	137.9
30	990	10.6	10.9	31.4%	31.8%	34.6	34.2
35	989	-1.6	-1.6	29.8%	30.2%	-5.3	-5.3
40	985	2.5	2.5	30.2%	30.6%	8.4	8.3
45	980	2.1	2.1	30.0%	30.3%	7.1	7.0
50	971	1.6	1.6	29.9%	30.3%	5.2	5.1
55	957	-3.2	-3.2	30.1%	30.5%	-10.5	-10.4
60	936	-3.8	-3.7	31.3%	31.7%	-11.8	-11.6
65	909	-1.1	-1.1	32.9%	33.3%	-3.2	-3.2
70	864	-2.0	-1.9	34.8%	35.2%	-5.5	-5.5
75	792	-4.0	-3.6	38.3%	38.8%	-9.5	-9.4
80	669	-6.5	-5.5	46.4%	46.8%	-11.8	-11.7
85	472	-12.1	-9.0	68.3%	68.5%	-13.2	-13.2
90	252	-12.1	-7.7	139.7%	136.2%	-5.5	-5.6
95	77	-12.1	-6.7	524.9%	475.3%	-1.3	-1.4
100	11			4715.5%	3481.9%		
Total:		**946**	**1244**			**2085**	**2074**

Source: Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.

Fig. 1 presents results of estimating *CNM* for selected European countries and regions and also for the USA from data on female population in 2004 (countries and regions are ranked according to their *Combined Fertility* in 2004; see further down for explanation of the concept). Mortality is taken into account, although assuming no mortality (and ignoring migration at ages above 80) yields similar results. Data for EU countries are estimated from Eurostat projection scenarios[6] (http://epp.eurostat.ec.europa.eu). For other countries, data are retrieved from web sites of national statistical offices.

For most of the economically developed countries, about 200-400 net migrants per 1000 original births in the cohort join the population cohort by the end of the life span. Most eastern EU countries have a negative *Completed Net Migration*, with up to 200 out of 1000 'indigenous inhabitants' (as is the case for Bulgaria) leaving the population by the end of life span. Some countries (Spain, Cyprus, Luxemburg, Italy, Malta and Switzerland) have enormous *Completed Net Migration*, with net migrants even outnumbering (in the case of Spain) the initial birth cohort by the end of the life span if rates observed in 2004 are continued.

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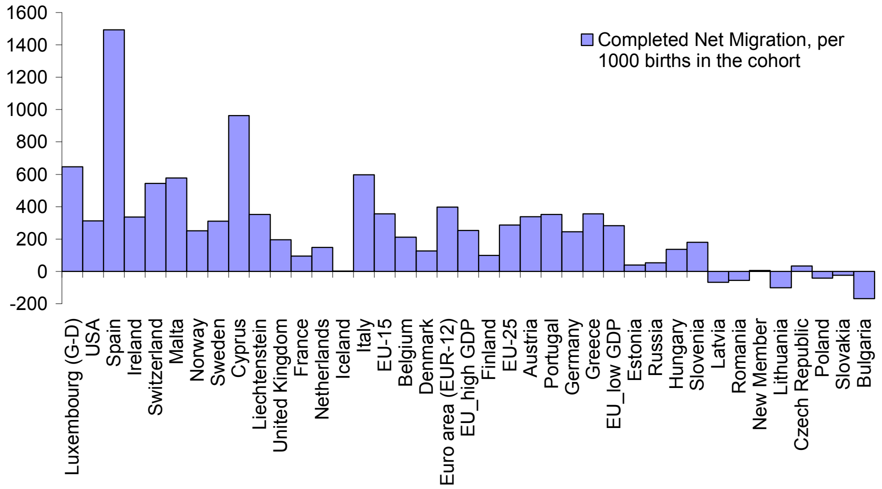
[6] At least, in the case of migration to Austria, the Eurostat scenario for net migration in 2004 was significantly lower compared to official estimates (see Appendix). For the sake of comparability, however, we present results based on Eurostat scenarios for all EU countries.

Figure 1 *Completed Net Migration* to selected countries and regions, females, 2004.



Source: Data for EU countries are estimated from Eurostat projection scenarios (http://epp.eurostat.ec.europa.eu). For other countries data are retrieved from web sites of national statistical offices.

3. MIGRATION AS A FACTOR OF FERTILITY AND POPULATION REPRODUCTION

Completed Net Migration might be an important concept, as it makes more transparent the impact of observed migration patterns on experience of population cohorts throughout their life course. However, this indicator says nothing about the role of migration in population *reproduction*. It does not help in addressing questions such as: Is the combination of net migration and of fertility enough to prevent the population from declining in the long-run? How long will it take before the descendants of the net immigrants outnumber the descendants of the indigenous population? What is the stable age structure implied by the observed combination of fertility, mortality and migration? What is the eventual effect of net migration on population ageing?

To make concrete the problems to be solved before properly addressing the questions that have arisen, one may look into the example of ten new EU Member States (all new member states except Romania and Bulgaria). Although its positive *CNM* might suggest a positive role of net migration for future population dynamics, consideration of the age structure of net migration rules out such a simplistic conclusion, see Fig. 2. Net migration—negative at childbearing ages—may in fact depress fertility and add to the decline of birth cohorts. Similarly, net migration which is positive at young ages and negative at old ages may boost fertility and result in a growth of birth cohorts.

Constant-rate migration at old ages may inflate population *size*. It may not, however, improve the reproduction *regimen* of the population. In particular, it may not protect the low-fertility population from depopulation. Migration at young ages, on contrary, may add to births and, therefore, improve the reproductive regimen. In order to quantify the effect of migration on population reproduction, one may look into the effect of migration on prospective sizes of birth cohorts in the population. In addition to the age pattern and the magnitude of migration rates, this effect depends on the fertility of net migrants.

This work introduces *Combined Fertility* (*CF*) as a measure of fertility for populations experiencing a sustainable level of migration; taking into account both the births from persons originally born in the population and births from net migrants. Fertility of those originally born in the population is denoted as *Original Fertility (OF)* and refers to a fertility level which might be observed in the absence of migration (we approximate it by traditional *TF*[7]). The (positive or negative) additive to fertility that is due to births from net migrants is called *Migratory Fertility* (*MF*) and measures the

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[7] This approximation may work in the case of no significant impact of former migrants on observed fertility rates. However, if the population experiences high migration and migrants' fertility is significantly different from that of non-migrants, *Original Fertility* is to be estimated separately, e.g., by approximating it by *TF* of local citizens.

stake of migration in the population reproduction. Hence, we denote by the *Combined Fertility* the sum of the total number of births produced by a person originally born into a birth cohort during the life course (i.e., *OF*, which is approximately equal to the conventional *TF*) *plus* the total number of births given to the population by immigrants joining the *same* cohort during the entire life of the cohort, per one baby born in the cohort (i.e., *MF*); both summands are calculated under some fixed age-specific migration and fertility rates and no mortality[8]:

$$CF = OF + MF \approx TF + MF\,. \tag{11}$$

As with the classic *TF*, *CF* reflects how effectively the population cohort is reproduced in the next generation. As in the classical case, a *CF* close to 2.1 is enough for simple replacement in a low-mortality population[9]. Unlike the classical case, however, we add migration to the picture of reproduction and thus use the concept of cohorts being *reproduced* by the fertility of both the original birth cohort and the migrants instead of being *reproduced* through the fertility of original cohort only. The concept is illustrated in Table 5 which presents calculations for a scenario of an indigenous population having a *TF* of 1.5 and net migrants adding another 0.6 births, hence bringing the population close to simple replacement. In

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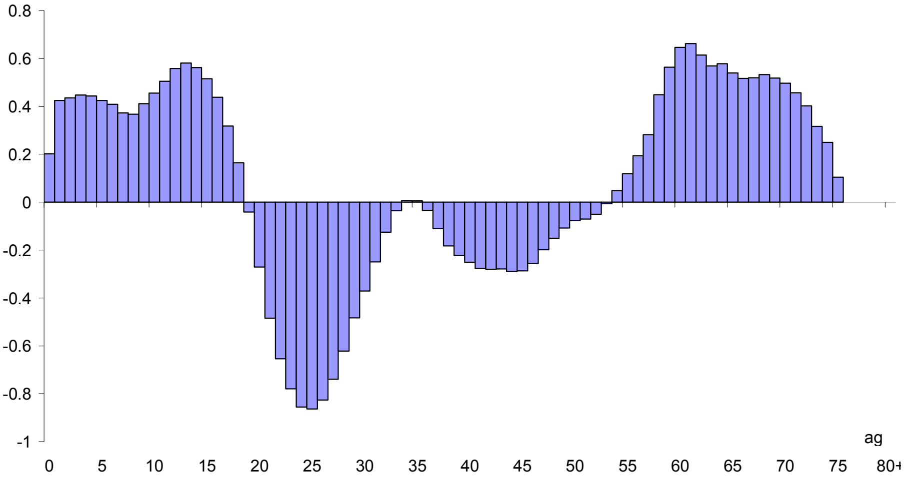
[8] However, we do imply mortality when estimating age-specific rates ${}_{\Delta}\delta_x$ of net migrants joining the original cohort, see *CNM* estimating procedures. Mortality is ignored in estimating the number of births from both indigenous inhabitants and migrants. Mortality and sex composition of births may also be put into the picture of cohort replacement, yielding *Combined Gross (Net) Reproduction* as a sum of *Original Gross (Net) Reproduction* and *Migratory Gross (Net) Reproduction*, the latter reflecting births *of own sex* given to the next generation by (*surviving)* migrants. Also, the measures of *Migratory* fertility and reproduction may be decomposed into summands linked to different migration flows and also to describe the impact of non-migratory population movements (generation-to-generation development of HIV prevalence might be an important example).

[9] Simple replacement is a regimen without generation-to-generation growth or decline of birth cohorts. This does not necessarily imply constant population size and not even constant number of births per year, as the latter depends on mean age at childbearing (Ediev 2005, 2007) and also on initial population structure.

other words, *Combined Fertility* of the population cohort is estimated as about 2.1, and it consists of the indigenous inhabitants' *Total Fertility* (1.5) and of *Migratory Fertility* due to children from migrants to join the cohort (0.6).

Table 6 presents results for Vienna (see migration details in Table 3), with net migrants fertility arbitrarily approximated by Austrian fertility in 1972. Not surprisingly, high net migration combined with high fertility assumption for net migrants result in grotesque addition of births from net migrants (more than double the births from the population cohort itself). As a result, Viennese birth cohorts seem to have no problem with replacement in the future, although about two-thirds of every subsequent generation is computed to be born from net migrants who have joined the preceding population cohort.

Figure 2 Age-specific net migration rates for 10 New Member States of EU, females, 2004.



Source: Data are estimated from Eurostat projection scenarios (http://epp.eurostat.ec.europa.eu).

Table 5 Calculations of *Original Fertility, Migratory Fertility*, and of *Combined Fertility* for an illustrative scenario (see Table 1 for description of the migration scenario).

Age, *x*	**Original birth cohort**	**Age-specific fertility rate, per 1000 aboriginals**	**Indigenous births at ages *x* to *x*+5**	**Cumulated number of net migrants joined at ages 0 to *x***	**Net migrants' person-years at ages x to x+5**	**Age-specific fertility rate of net migrants, per 1000**	**Births from net migrants at ages *x* to *x*+5**	**Total number of births at ages x to x+5**
(*x*)	**(1)**	**(2)**	**(3)= 5*(1)*(2) /1000**	**(4)**	**(5)**	**(6)**	**(7) =(5)*(6) /1000**	**(8) = (3)+(7)**
0	1000			0	125			
5	1000			50	375			
10	1000			100	625			
15	1000	10	50	150	875	13	11.4	61.4
20	1000	40	200	200	1125	60	67.5	267.5
25	1000	80	400	250	1375	80	110.0	510.0
30	1000	80	400	300	1625	90	146.3	546.3
35	1000	60	300	350	1875	90	168.8	468.8
40	1000	20	100	400	2000	30	60.0	160.0
45	1000	10	50	400	2000	18	36.0	86.0
50	1000			400	2000			
55	1000			400	2000			
60	1000			400	2000			
65	1000			400	2000			
70	1000			400	2000			
75	1000			400	2000			
80	1000			400	2000			
85	1000			400	2000			
90	1000			400	2000			
95	1000			400	2000			
100	1000			400	-			
	Totals:	**1.500**[a]	**1500**			**1.905**[a]	**599.9**	**2099.9**

[a] *Total Fertility* obtained as the sum of age-specific fertility rates multiplied by duration of the age interval (i.e., by five).

Table 6 Calculations of Original Fertility, Migratory Fertility, and of Combined Fertility for an illustrative scenario (Vienna, females, 2004; migrants fertility is set at arbitrary level).

Age, *x*	**Original birth cohort**	**Age-specific fertility rate, per 1000 aborigi-nals**	**Indigenous births at ages *x* to *x*+5**	**Cumulated number of net migrants joined at ages 0 to *x***	**Net migrants' person-years at ages x to x+5**	**Age-specific fertility rate of net migrants, per 1000**	**Births from net migrants at ages *x* to *x*+5**	**Total number of births at ages x to x+5**
(*x*)	**(1)**	**(2)**	**(3)= 5*(1)*(2) /1000**	**(4)**	**(5)**	**(6)**	**(7) =(5)*(6) /1000**	**(8)= (3)+(7)**
0	1000			0	139			
5	1000			56	407			
10	1000	0.2	1	107	688			1
15	1000	22.2	111	168	1875	55.7	104	215
20	1000	62.2	311	582	5205	141.2	735	1046
25	1000	80.7	403	1500	9243	105.4	974	1377
30	1000	73.5	367	2197	11418	65.7	750	1118
35	1000	37.6	188	2370	11785	36.1	426	614
40	1000	7.6	38	2344	11823	10.9	129	167
45	1000	0.5	3	2386	12017	0.6	7	10
50	1000			2421	12170			
55	1000			2447	12104			
60	1000			2394	11825			
65	1000			2336	11637			
70	1000			2319	11528			
75	1000			2292	11340			
80	1000			2244	11073			
85	1000			2185	10760			
90	1000			2119	10526			
95	1000			2091	10441			
100	1000			2085	5213			
	Totals:	**1.423**[a]	**1423**			**2.078**[a]	**3125**	**4548**

[a] *Total Fertility* obtained as the sum of age-specific fertility rates multiplied by duration of the age interval (i.e., by five).

Source: Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005. Fertility profile of net migrants is set arbitrarily for illustration only.

4. Measurement

When it comes to estimate *Migratory Fertility* for real populations, we might not know exactly the age-specific fertility rates for migrants. In fact, fertility of migrants may depend not only on their age but also on duration of their stay in the population. Therefore, straightforward calculations presented should be replaced by other, more robust, estimation procedures. Such procedures may rely on the following relation between *Migratory Fertility* and prospective fertility of migrants:

$$MF = \Delta \cdot \sum_{x} {}_{\Delta}\delta_x \cdot PF^{m}{}_{x+\Delta/2} \ , \tag{12}$$

here $PF^{m}{}_{x}$ stands for prospective fertility of net migrants joined the cohort at age x.

Assuming all net migrants to have similar total fertility, we may rewrite Eq. (12) in the following form:

$$MF = TF^{m} \cdot \Delta \cdot \sum_{x} {}_{\Delta}\delta_x \cdot \frac{PF^{m}{}_{x+\Delta/2}}{TF^{m}} \ . \tag{13}$$

This expression is more robust, as it relies on the total fertility of migrants and on the *proportion* of the prospective fertility by age which should be less variable than the age-specific fertility rates themselves. The latter expression in Eq. (13) may be interpreted as a product of migrants' total fertility by the *corrected* number of net migrants to join the cohort. This point is important, as below we will address alternative approaches for correcting the migration statistics with the same purpose of estimating the *MF*. Table 7 illustrates usage of Eq. (13). To illustrate the robustness of the method, corrections of the net migration rates are obtained both from the assumed fertility profile of migrants themselves and from the fertility profile implied for indigenous inhabitants. Usage of indigenous fertility instead of migrants' fertility results only in moderate differences in the *Migratory* and *Combined Fertility* estimates. In the case of Vienna the difference between exact and approximate estimates was considerably wider (see Table 8).

Table 7 Calculations of *Migratory Fertility* and of *Combined Fertility* for an illustrative scenario (see Tables 1, 4 for description of the migration and fertility scenarios).

Age	Original birth cohort	Adjusted age-specific net migration rate, per 1000 cohort births	Age-specific fertility rate, per 1000		Proportion of prospective fertility at exact age x		Corrected age-specific net migration, per 1000 cohort births	
			migrants	indigenous	migrants	indigenous	exact	approximate
(x)	(1)	(2)	(3)	(4)	(5)	(6)	(7) =(2)*(5)	(8) =(2)*(6)
0	1000	10			100.0%	100.0%	10.00	10.00
5	1000	10			100.0%	100.0%	10.00	10.00
10	1000	10			100.0%	100.0%	10.00	10.00
15	1000	10	13	10	98.3%	98.3%	9.83	9.83
20	1000	10	60	40	88.7%	90.0%	8.87	9.00
25	1000	10	80	80	70.3%	70.0%	7.03	7.00
30	1000	10	90	80	48.0%	43.3%	4.80	4.33
35	1000	10	90	60	24.4%	20.0%	2.44	2.00
40	1000		30	20	8.7%	6.7%		
45	1000		18	10	2.4%	1.7%		
50	1000				0.0%	0.0%		
55	1000				0.0%	0.0%		
60	1000				0.0%	0.0%		
65	1000				0.0%	0.0%		
70	1000				0.0%	0.0%		
75	1000				0.0%	0.0%		
80	1000				0.0%	0.0%		
85	1000				0.0%	0.0%		
90	1000				0.0%	0.0%		
95	1000				100.0%	100.0%		
100	1000				100.0%	100.0%		
Total		**400.0**	**1.905**[a]	**1.500**[a]			**314.9**	**310.8**
						$\mathit{Migratory\ Fertility}=\mathit{TM}*\mathit{TF}^{m}$:	**599.9**	**592.1**
						$\mathit{Combined\ Fertility}=1000*\mathit{OF}+\mathit{MF}$:	**2099.9**	**2092.1**

[a] *Total Fertility* obtained as the sum of age-specific fertility rates multiplied by duration of the age interval (i.e., by five).

Table 8 Calculations of *Migratory Fertility* and of *Combined Fertility* for an illustrative scenario (Vienna, females, 2004; see Tables 2 and 6 for description of the migration and fertility scenarios).

Age	**Original birth cohort**	**Adjusted age-specific net migration rate, per 1000 cohort births**	**Age-specific fertility rate, per 1000**		**Proportion of prospective fertility at exact age x**		**Corrected age-specific net migration, per 1000 cohort births**	
			migrants	indigenous	migrants	indigenous	exact	approximate
(x)	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7) =(2)*(5)**	**(8) =(2)*(6)**
0	1000	11.2			100.0%	100.0%	11.2	11.2
5	1000	10.3			100.0%	100.0%	10.3	10.3
10	1000	12.2	0	0.20	100.0%	100.0%	12.2	12.2
15	1000	82.8	55.69	22.19	93.3%	96.0%	77.3	79.5
20	1000	183.6	141.18	62.20	69.6%	81.2%	127.8	149.1
25	1000	139.4	105.36	80.70	40.0%	56.1%	55.7	78.2
30	1000	34.6	65.7	73.49	19.4%	29.0%	6.7	10.0
35	1000	-5.3	36.11	37.60	7.1%	9.5%	-0.4	-0.5
40	1000	8.4	10.92	7.62	1.5%	1.5%	0.1	0.1
45	1000	7.1	0.62	0.53	0.1%	0.1%	0.0	0.0
50	1000	5.2			0.0%	0.0%	0.0	0.0
55	1000	-10.5			0.0%	0.0%	0.0	0.0
60	1000	-11.8			0.0%	0.0%	0.0	0.0
65	1000	-3.2			0.0%	0.0%	0.0	0.0
70	1000	-5.5			0.0%	0.0%	0.0	0.0
75	1000	-9.5			0.0%	0.0%	0.0	0.0
80	1000	-11.8			0.0%	0.0%	0.0	0.0
85	1000	-13.2			0.0%	0.0%	0.0	0.0
90	1000	-5.5			0.0%	0.0%	0.0	0.0
95	1000	-1.3			0.0%	0.0%	0.0	0.0
100	1000				0.0%	0.0%		
Total		**2085**	**2.078**[a]	**1.423**[a]			**1504**	**1750**
						___Migratory Fertility=Total*TF^{m}:___	**3125**	**3637**
						___Combined Fertility =1000*OF+MF:___	**4548**	**5060**

[a] *Total Fertility* obtained as the sum of age-specific fertility rates multiplied by duration of the age interval (i.e., by five).

Source: Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.
Fertility profile of net migrants is set arbitrarily for illustration only.

Calculations presented in Tables 5-8 utilise ideas underlying several interrelated concepts: reproductive value of Fisher (1930), demographic potential as a generalisation of the reproductive value (Ediev 2001, 2003), and tightly linked concepts of reproductive worth (Tognetti 1976) and of genealogical lines (Ediev 2005)[10]. These concepts focus on the prospective fertility of persons or—to put it into a framework of genealogical lines—on the prospective number of direct descendants. Apart from being dependent on approximations for migrants' age-specific fertility, these concepts, being applied to migration in their simplest form, assume no important links between migration and fertility. It is in particular important that migration of children is apparently linked to the migration of their parents. On the other hand, the prospective fertility of migrants is apparently linked to the number of children they had prior to migration, i.e., to migration at young ages. For these reasons, one might look into a modification of the procedure for estimating the impact of net migration on population reproduction.

The concept remains the same, but the estimation procedure is somewhat different from that which was presented above. The idea is to estimate the *Migratory Fertility* as a product of migrants' *TF* by the *effective* number of net migrants who bring all their children to the population, considering both future births and children who migrated with their parents as additional to the births to the next generation, rather than to the cohort under study itself. The advantage of this approach is that it is much easier and more robust to estimate migrants' *TF* compared to estimating the age pattern of their fertility rates. Besides, migrants belonging to distinct generations will not be counted in this approach as joining the same cohort of the receiving population. In the illustrative example presented, migrants own *TF* is set to be 1.91, somewhat higher than that of the indigenous population. There are 400 net migrants who will ultimately join the

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[10] In fact, *prospective fertility* equals to *reproductive worth* by Tognetty or to prospective number of *genealogical lines* connected to the person, under no mortality. Also, it equals to *reproductive value* and *demographic potential* of the person if simple replacement regimen is concerned.

population cohort. Were they all to have a *TF* of 1.91, there would be $400 \cdot 1.91 = 762$ additional births. This figure is higher than the actual number of additional births presented in the table (about 600) due to the fact that some of net migrants arrive in the population in middle age, already having had some children and their prospective fertility is, thereby, to be accordingly discounted. To put into another context, some migrants arrive being children of other migrants and, therefore, are counted twice in the aforementioned simplistic estimate: both as migrants and as children born from migrants. Ignorance of this fact results in an overestimation of migrants' descendants due to a double counting of children who migrate following their parents. To avoid this double-counting, age-specific net migration must be corrected in a way that excludes counting both parental and child generations together. Indeed, Eq. (13) already presents an example of such 'correction', as net migrants are counted only in proportions related to their expected prospective fertility. That particular correction, however, is somewhat opposite to the correction that we are looking for. Instead of removing the children who migrate following their parents from migration statistics, it is the parental generation that is corrected in Eq. (13)—comparing the last and the third (col. (2)) columns in Tables 7 and 8 reveals this point quite explicitly. In order to find a better correction procedure—and assuming for the sake of argument that the migrants' age-specific fertility is known—one might start by deducting the migrants' children from net migration statistics:

$$ {}_{\Delta}M^{c}{}_{x} = {}_{\Delta}M_{x} - \gamma \cdot \Delta \cdot \sum_{y \geq x} \frac{{}_{\Delta}M_{y} + {}_{\Delta}M_{y+\Delta}}{2} \, {}_{\Delta}F^{m}{}_{y-x} , \qquad (14) $$

here ${}_{\Delta}M^{c}{}_{x}$ stands for corrected net migration at ages x to $x + \Delta$, ${}_{\Delta}M_{\omega} = 0$ (ω is maximum life span, e.g., 100), and γ is the share of own-sex births (in all our calculations we suppose the sex ratio at birth to be 1.055 males to female)[11]. Mortality is not included in (14) as we develop the *TF* concept which is obtained implying no mortality; besides, corrections (14) are more important for ages with low mortality.

In Eq. (14) net migrants at all age groups are deducted by number of same-age and same-sex children of elder net migrants. Indeed, results of the correction depend on fertility assumption used in (14). This is illustrated on Fig. 3 which presents age profiles of net migration for Vienna, both original and corrected. Two alternative corrections are presented on the figure: obtained by applying fertility rates of the population of Vienna, 2004, and of the population of Austria in 1972. Seemingly, both fertility profiles used in the corrections are not in accordance with actual fertility of net migrants to Vienna in 2004. This may be noted from the fact that the correction did not set to zero net migration of children. Under proper correction—assuming a negligible number of children migrating without their parents—one should obtain no net migration at youngest age groups. In particular, profiles of corrected net migration presented on Fig. 3 suggest that fertility of net migrants in years preceding the migration is lower compared to fertility patterns of general population (this is seen from the fact that corrected numbers for ages 0-5 are negative and significantly lower compared to the

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[11] In case of $\gamma \cdot TF = 1$ correction procedure (14) is equivalent to (13). Indeed, one may rewrite the sum in (13) as follows:

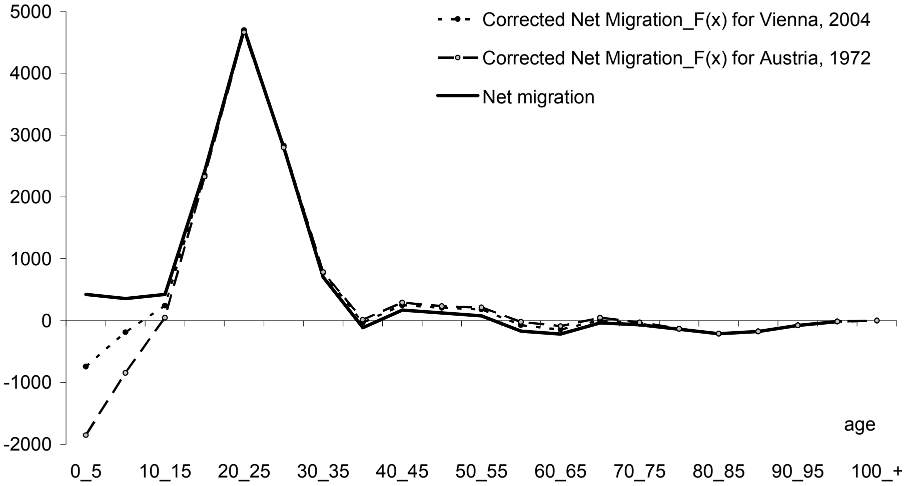
$$\sum_x {}_\Delta\delta_x \cdot \left(1 - \frac{RF^m{}_{x+\Delta/2}}{TF^m}\right) = \sum_x {}_\Delta\delta_x - \sum_x {}_\Delta\delta_x \cdot \left(\frac{\Delta}{2}\frac{{}_\Delta F^m{}_x}{TF^m} + \sum_{y<x}\frac{\Delta\cdot{}_\Delta F^m{}_y}{TF^m}\right) =$$

$$= \sum_x \left({}_\Delta\delta_x - \frac{\Delta}{TF^m}\sum_{y\geq x}\frac{{}_\Delta\delta_y + {}_\Delta\delta_{y+\Delta}}{2}\,{}_\Delta F^m{}_{y-x}\right)$$

, here *RF* stands for retrospective fertility (*RF*=*TF*-*PF*). Summands in the last expression are arranged in such a way that all summands in parentheses correspond to migrants or to same-age children of elder migrants. In our derivations we ignore mortality. Would the mortality be counted, corrections would include survival probabilities, the condition addressed here would refer to simple replacement (*NR*=1), and the concepts addressed would be even closer to the concepts of reproductive worth, genealogical lines, reproductive value, and demographic potential.

next two age groups). Partially, the patterns of corrected net migrations may also be explained by children migrating several years after the migration of their parents (this will depress observed migration at youngest ages and boost migration of elder children).

Figure 3 Age-specific net migration rates to Vienna, females, 2004: original data and two alternative corrections based on different fertility scenarios.



Source: Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.

The problems listed above might result in biases in correcting net migration by (14). These problems may be avoided, however, if one introduces a threshold 'age of *maturity*' $x_{\min}$, defined so that those under this age $x_{\min}$ are all children of other migrants and their migration is a direct consequence of the migration of their parents (importantly, they may actually migrate several years after their parents; yet their migration is still considered to follow the migration of their parents). The age of maturity may be suggested by the distribution of migrants by age. Conveniently, see, e.g., Fig. 3, one may set $x_{\min} = 15$ as net migration starts growing rapidly at ages above 15 only. Hence, simply ignoring all net migrants of ages below $x_{\min}$

might be a more robust alternative to correcting procedure (14). At elder ages, however, some correction procedure is still to be applied as some migrants may potentially migrate with children at ages above the maturity age. Even more importantly for migration modelling, parents may migrate *following* their mature children. If the first situation is assumed to prevail, one should apply correction (14) to migrants of age higher than $x_{\min}$. However, if the second situation prevails—and this assumption seems to be more reasonable as most migrants migrate at young working/studying/marrying ages—then it is better to deduct parents rather than children from net migration statistics at ages above $x_{\min}$. In fact, both these approaches produce very similar results when—as is mostly the case in reality—net migration at old ages is very low. Following the second approach, one may develop quite a robust correction procedure similar to what was presented for correcting the migration at young ages. One may simply ignore net migration statistics at ages above some maximal age $x_{\max}$, assuming number of migrants above that age to be a good approximation to the number of parents migrating following their children. This simple method is also facilitated by the fact that fertility at old ages (say, above 50 or 45) is negligible and, hence, ignoring migrants at old ages should not result in any biased conclusions on the effects of migration upon births in the population.

Therefore, a simple robust option for correcting the net migration might be to abandon migration statistics at ages below some minimal age $x_{\min}$ (to approximately eliminate the children population from statistics) and above some maximal age $x_{\max}$ (to eliminate parents of middle-age migrants from statistics):

$$ {}_{\Delta}M^{c}{}_{x} = \begin{cases} 0, & x < x_{\min} \\ {}_{\Delta}M_{x}, & x_{\min} \le x \le x_{\max} \\ 0, & x > x_{\max} \end{cases} \tag{15} $$

We mostly rely on method (15) in correcting migration statistics with $x_{\min} = 15$ and $x_{\max} = 44$ both for females and males. Other methods presented are used to check the robustness and reliability of estimates.

The maximal and minimal ages in (15) may be derived in a systematic way, based on still another correcting method. The idea is to weight older ages instead of ignoring the net migration at these ages. The rational behind such weighing is similar to that used in prospective-fertility-based corrections (13). It is based on estimating proportions of children of age below the minimal age $x_{\min}$ or still to be born at the year of their parent's migration. Assuming that those of age equal to or above $x_{\min}$ take their own decision according migration, we should subtract them from the number of children added to the receiving population by their parents. If these children (of age equal to or above $x_{\min}$) do, in fact, also immigrate, they will already be counted by the migration statistics, i.e., subtracting them from their parents total fertility would eliminate double counting. Let us denote by c_x the proportion of children younger than $x_{\min}$ (or still to be born) for migrants of ages *x* to $x + \Delta$:

$$ {}_{\Delta}c_x = \begin{cases} 1, \quad x < x_{\min} \\ \dfrac{\Delta}{TFR^m} \cdot \sum\limits_{y=x-x\min}^{\omega} \dfrac{{}_{\Delta}F^m{}_y + {}_{\Delta}F^m{}_{y+\Delta}}{2}, \ x \geq x_{\min} \end{cases} \tag{16} $$

Using these proportions, migration numbers by age may be corrected in the following way:

$$ {}_{\Delta}M^c{}_x = \begin{cases} 0, \ x < x_{\min} \\ {}_{\Delta}c_x \cdot {}_{\Delta}M_x, \ x \geq x_{\min} \end{cases} \tag{17} $$

This approach is slightly more complicated than correction (15). However, it is more transparent and, in fact, it may be used to derive explicitly the maximal age $x_{\max}$ in (15). To do so, one should equate total corrected numbers of net migrants obtained by both procedures:

$$\sum_{x=x\min}^{x\max} {}_\Delta M_x = \sum_{x=x\min}^{\omega} {}_\Delta c_x \cdot {}_\Delta M_x \ . \tag{18}$$

Solving this equation for $x_{\max}$, one may explicitly derive the optimal maximal age, above which net migration statistics is to be ignored in studying the Migratory Fertility[12]. Based on real data, with $x_{\min} = 15$, we have obtained $x_{\max}$ about 42-44 for both females and males from Eq. (18).

Using corrected migration by age, one may calculate corrected migration rates:

$$_\Delta m^c{}_x = \frac{_\Delta M^c{}_x}{\Delta \cdot \left(\frac{1}{2}\left(1 + {}_\Delta p_x\right) \cdot N_x + \frac{1}{2}{}_\Delta M_x\right)} . \tag{19}$$

Note that here we again divide corrected net migration by population person-years, including uncorrected net migration in the denominator.

Net migration rates per population at the beginning of the age interval and per births in the indigenous cohort are derived in the same manner, as was presented above:

$$_\Delta \pi^c{}_x = \tfrac{1}{2}\left(1 + {}_\Delta p_x\right) \frac{_\Delta m^c{}_x}{1 - \frac{1}{2}\Delta \cdot {}_\Delta m_x} , \tag{20}$$

$$\begin{aligned} _\Delta \delta^c{}_x &= \frac{_\Delta M^c{}_x}{\Delta \cdot B} = \frac{_\Delta M^c{}_x}{\Delta \cdot \nu^{ab}{}_x P_x} = \frac{_\Delta \pi^c{}_x}{\nu^{ab}{}_x} \\ &= \frac{\frac{1}{2}\left(1 + {}_\Delta p_x\right)}{\nu^{ab}{}_x \left(1 - \frac{1}{2}\Delta \cdot {}_\Delta m_x\right)} {}_\Delta m^c{}_x . \end{aligned} \tag{21}$$

Again, note that these are uncorrected net migration rates which

---

[12] For migration of nearly the same size at all age groups this implies

$$x_{\max} = x_{\min} + \sum_{x=x\min}^{\omega} {}_\Delta c_x - 1 .$$

Comparison of this simplified expression to direct estimates from (18) based on real data show that the difference between these two estimates is about 2-3 years only, with the simplified expression leading to overestimates compared to (18).

enter the denominator. For the sake of simplicity, we keep unchanged the formulas for the proportion of original births in the total population, see. Eq. (3)-(5), (8). Nonetheless, it is clear that these formulas are only approximate, as the migration at young ages is no longer assumed to follow observed rates independently of migration at elder ages.

Summing up the age-specific 'cohort joining' rates (21), yields effective number of net migrants who join the cohort and add all their children to births in the next generation:

$$
\begin{aligned}
EM &= \Delta \cdot \sum_x {}_\Delta\delta^c{}_x = \Delta \cdot \sum_x \frac{{}_\Delta\pi^c{}_x}{\nu^{ab}{}_x} \\
&= \Delta \cdot \sum_x \frac{\frac{1}{2}\left(1 + {}_\Delta p_x\right)}{\nu^{ab}{}_x\left(1 - \frac{1}{2}\Delta \cdot {}_\Delta m_x\right)} {}_\Delta m^c{}_x \; . \qquad (22)
\end{aligned}
$$

Here *EM* stands for '*Effective Migration*', i.e., the effective number of net migrants who add *all* their fertility to the population.

Using exponential approximation (8), one may also derive:

$$
EM \approx \Delta \cdot \sum_x \frac{\frac{1}{2}\left(1 + {}_\Delta p_x\right)_x p_0 \cdot e^{\Delta \sum_{y=0}^{x-\Delta} {}_\Delta m_y}}{1 - \frac{1}{2}\Delta \cdot {}_\Delta m_x} \cdot {}_\Delta m^c{}_x \; . \qquad (23)
$$

Finally, the *MF* equals the product of *EM* and migrants' *TF*, and the *CF* may be calculated from (1):

$$
CF = OF + MF \approx TF + TF^m \cdot EM \; . \qquad (24)
$$

An advantageous feature of this approach is that one may simply suppose the migrants total fertility to be proportional to that of non-migrants, $TF^m = \alpha \cdot OF$, and use the following approximation to (16) without getting into estimations of age patterns of migrants fertility:

$$
CF \approx TF \cdot \left(1 + \alpha \cdot EM\right). \qquad (25)
$$

Note that the age profile of migrants' fertility may be completely different from that of non-migrants. However, this will not affect the accuracy of our formula as long as corrections of migration statistics for double-counting of children and estimates of the total fertility of migrants are accurate.

Table 9 Corrected age-specific net migration rates, per 1000 original cohort births calculated from different estimation procedures. Vienna, females, 2004 (see Tables 2 and 5 for description of the scenarios).

Age (*x*)	**Correction (13) using fertility profile of**		**Correction (14) using fertility profile of**		**Corr. (14) with ignoring migration at ages below 15, fertility profile of**		**Correction (15)**	**Correction (17) using fertility profile of**	
	migrants	indigenous	migrants	indigenous	migrants	indigenous		migrants	indigenous
0	11.2	11.2	-49.1	-19.7					
5	10.3	10.3	-24.3	-5.4					
10	12.2	12.2	1.3	7.0					
15	77.3	79.5	79.6	80.8	79.6	80.8	82.8	82.8	82.8
20	127.8	149.1	181.6	183.0	181.6	183.0	183.6	183.6	183.6
25	55.7	78.2	139.5	141.1	139.5	141.1	139.4	139.4	139.4
30	6.7	10.0	38.6	38.1	38.6	38.1	34.6	32.3	33.3
35	-0.4	-0.5	0.6	-1.6	0.6	-1.6	-5.3	-3.7	-4.3
40	0.1	0.1	14.5	12.3	14.5	12.3	8.4	3.4	4.7
45	0.0	0.0	13.5	12.2	13.5	12.2		1.4	2.1
50	0.0	0.0	13.9	11.8	13.9	11.8		0.4	0.5
55	0.0	0.0	-1.3	-4.5	-1.3	-4.5		-0.2	-0.2
60	0.0	0.0	-4.7	-7.9	-4.7	-7.9		0.0	0.0
65	0.0	0.0	4.1	0.3	4.1	0.3		0.0	0.0
70	0.0	0.0	-2.6	-4.2	-2.6	-4.2		0.0	0.0
75	0.0	0.0	-8.9	-9.3	-8.9	-9.3		0.0	0.0
80	0.0	0.0	-11.7	-11.8	-11.7	-11.8		0.0	0.0
85	0.0	0.0	-13.2	-13.2	-13.2	-13.2		0.0	0.0
90	0.0	0.0	-5.5	-5.5	-5.5	-5.5		0.0	0.0
95	0.0	0.0	-1.3	-1.3	-1.3	-1.3		0.0	0.0
100	-	-	-	-	-	-	-	-	-
***EM*[a]**	**1504**	**1750**	**1823**	**2011**	**2184**	**2102**	**2218**	**2197**	**2209**
***Scenario 1 (Original Fertility (OF)=1423, Migrants TF=2078)*[b]:**									
MF	**3125**	**3637**	**3788**	**4180**	**4537**	**4368**	**4608**	**4565**	**4590**
CF	**4548**	**5060**	**5211**	**5602**	**5960**	**5790**	**6031**	**5988**	**6013**
***Scenario 2 (Original Fertility (OF)=1423, Migrants TF=1423)*[b]:**									
MF	**2140**	**2490**	**2594**	**2862**	**3106**	**2990**	**3155**	**3125**	**3143**
CF	**3563**	**3913**	**4016**	**4284**	**4529**	**4413**	**4578**	**4548**	**4565**

[a] ***Effective Migration*** is the sum of corrected age-specific net migration rates multiplied by five

[b] ***Migratory Fertility (MF)=Migrants TF*EM; Combined Fertility (CF)=OF+MF***

Source: Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.

Table 10 Corrected age-specific net migration rates, per 1000 original cohort births calculated from different estimation procedures. Vienna, females, 2004 (see Tables 2 and 5 for the scenarios). No mortality is implied.

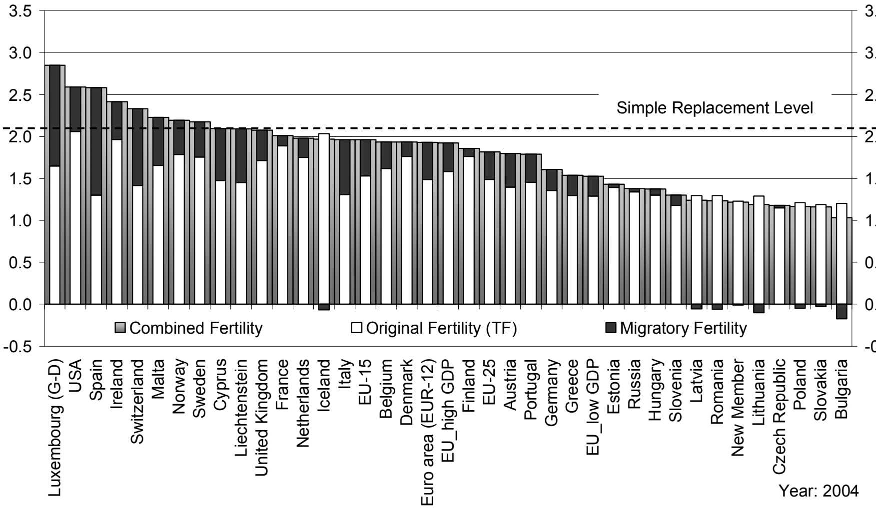
Age (*x*)	**Correction (13) using fertility profile of**		**Correction (14) using fertility profile of**		**Corr. (14) with ignoring migration at ages below 15, fertility profile of**		**Correction (15)**	**Correction (17) using fertility profile of**	
	migrants	indigenous	migrants	indigenous	migrants	indigenous		migrants	indigenous
0	11.2	11.2	-49.2	-19.7					
5	10.3	10.3	-24.4	-5.4					
10	12.2	12.2	1.3	7.0					
15	77.7	80.0	80.1	81.3	80.1	81.3	83.3	83.3	83.3
20	128.7	150.1	182.9	184.3	182.9	184.3	184.9	184.9	184.9
25	56.2	78.8	140.7	142.3	140.7	142.3	140.6	140.6	140.5
30	6.8	10.1	39.0	38.5	39.0	38.5	35.0	32.6	33.6
35	-0.4	-0.5	0.7	-1.7	0.7	-1.7	-5.4	-3.8	-4.4
40	0.1	0.1	14.7	12.6	14.7	12.6	8.6	3.4	4.8
45	0.0	0.0	13.9	12.5	13.9	12.5		1.4	2.1
50	0.0	0.0	14.4	12.3	14.4	12.3		0.4	0.5
55	0.0	0.0	-1.4	-4.7	-1.4	-4.7		-0.2	-0.2
60	0.0	0.0	-5.1	-8.5	-5.1	-8.5		0.0	0.0
65	0.0	0.0	4.6	0.3	4.6	0.3		0.0	0.0
70	0.0	0.0	-3.1	-5.1	-3.1	-5.1		0.0	0.0
75	0.0	0.0	-12.2	-12.7	-12.2	-12.7		0.0	0.0
80	0.0	0.0	-20.6	-20.7	-20.6	-20.7		0.0	0.0
85	0.0	0.0	-36.8	-36.8	-36.8	-36.8		0.0	0.0
90	0.0	0.0	-34.7	-34.7	-34.7	-34.7		0.0	0.0
95	0.0	0.0	-32.6	-32.6	-32.6	-32.6		0.0	0.0
100	-	-	-	-	-	-	-	-	-
EM [a]	**1514**	**1762**	**1360**	**1541**	**1721**	**1632**	**2234**	**2213**	**2225**
Scenario 1 (Original Fertility (OF)=1423, Migrants TF=2078) [b]**:**									
MF	**3146**	**3661**	**2826**	**3202**	**3577**	**3391**	**4643**	**4599**	**4624**
CF	**4568**	**5084**	**4248**	**4625**	**4999**	**4814**	**6065**	**6021**	**6047**
Scenario 2 (Original Fertility (OF)=1423, Migrants TF=1423) [b]**:**									
MF	**2154**	**2507**	**1935**	**2193**	**2449**	**2322**	**3179**	**3149**	**3166**
CF	**3576**	**3929**	**3357**	**3615**	**3872**	**3744**	**4601**	**4571**	**4589**

[a] ***Effective Migration*** is the sum of corrected age-specific net migration rates multiplied by five

[b] ***Migratory Fertility (MF)=Migrants TF*EM; Combined Fertility (CF)=OF+MF***

Source: Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.

Figure 4 Estimates of Total Fertility, Migratory Fertility and Combined Fertility for female population of selected countries and regions in 2004.



Source: Data for EU countries are estimated from Eurostat projection scenarios[13] (http://epp.eurostat.ec.europa.eu). For other countries data are retrieved from web sites of national statistical offices.

Table 9 presents results of estimating *Effective Migration*, *Migratory Fertility* and *Combined Fertility* for Vienna using different procedures (13)-(15), (17) for correcting the net migration statistics and under different scenarios for net migrants' total fertility. Correction (13) underestimates the migratory impact on fertility due to underestimating the proportions of prospective fertility at those ages when net migration is most important. Apparently, migration causes postponement of births by migrants and therefore any application of fertility profiles from general populations to migrants will tend to underestimate their prospective fertility at young

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[13] At least, in case of migration to Austria, Eurostat scenario for net migration in 2004 was significantly lower compared to official estimates (see the Appendix). For the sake of comparability, however, we present results based on Eurostat scenarios for all EU countries.

childbearing ages. At the same time, migration is highest at these same ages and, therefore, corrections based on proportions of prospective fertility may lead to strong underestimation of migratory fertility.

Correction (14) is somewhat better, as it discounts numbers of children migrating rather than numbers of their parents. However, see discussion above, correction (14) is close to (13), especially for migrants *TF* close to simple replacement. Correction (14) is also heavily dependent on assumptions about fertility profile as it is seen from non-zero numbers of children migrants. This bias may be improved by ignoring correction results at ages below 15 and setting effective migration of children to be zero (as all these children are to be counted further on in total fertility of their parents)—this is done in columns (5) and (6) of the table. Remaining bias of correction (14) occurs when there is significant net migration at old ages—as it is in the case presented. The problem is that correction (14) and its modification implicitly assume that if a migrant of *any* age migrates to/from the population, so do all his children. As was argued before, this might not be the case for children old enough to make their own migration decisions. Because this fact was not taken into account there was significant underestimation of migratory fertility using (14) and its modification due to significant negative net migration at old ages. It is therefore not surprising that corrections (15) and (17), which rest on assuming that above age 15 everybody takes migration decisions independently, result in higher estimates of *Effective Migration* and of *Migratory Fertility*. It is of key importance for practical applications that the corrections (15) and (17) are less sensitive to mortality assumptions. If this is so, these methods provide robust estimates even when mortality is not considered at all stages of computations, including computation of age-specific cohort-joining rates. This is illustrated in Table 10 which presents the same results, as in Table 8, but with no mortality at all. Hence, corrections (15) and (17) are based on more consistent migration models and yield more robust results. Simplest truncating method (15) works very well and may robustly be implemented in practical calculations. Estimates based on this method for selected countries and regions are presented on Fig. 5.

5. IMPLICATIONS FOR THE POPULATION COMPOSITION BY ORIGIN

Completed Fertility and its components may also tell us explicitly about the dynamics of population composition. The descendants of migrants will constitute the share $\frac{MF}{CF}$ in the next generation; share of descendants of those originally born in the population will decrease by $\frac{OF}{CF} = 1 - \frac{MF}{CF}$ in every next generation.

Consider, for example, the population cohort with *TF* equal to 1.6 and with migrants adding another 0.5 births to the next generation per one baby of the cohort. In this example the *CF* equals 2.1, i.e., the population cohort will simply be replaced in the next generation, meaning neither sustainable growth nor decline of cohorts under continuation of the rates to the future. Share of births from those originally born in the population will be $\frac{1.6}{2.1} \approx 76\%$, i.e., each generation will face a decrease of the share of descendants of the original population by about 24% under imputed fertility and migration rates. The speed of replacing the initial population by descendants of immigrants may be put into simpler form, if we calculate the duration of time necessary to replace the birth cohort in half by descendants of migrants:

$$\tau_{0.5} = T \cdot \frac{\ln(1-0.5)}{\ln\left({}^{OF}\!/_{CF}\right)}, \tag{26}$$

where *T* is the generation length (around 27 years for human populations). In the example presented the time to half-replacement by migration would be about 69 years with generation length equal to 27 years.

It is very convenient that the time-to-half-replacement (26) may be estimated regardless of the fertility levels of the original and migrant populations, as long as we have estimates for the *Effective Migration* and for the ratio of the migrants TF to that of the original population (α): $\frac{OF}{CF}=\frac{OF}{OF+MF}=\frac{OF}{OF+\alpha\cdot OF\cdot EM}=\frac{1}{1+\alpha\cdot EM}$, i.e.,

$$\tau_{0.5}=T\cdot\frac{\ln(1-0.5)}{\ln\left(1/(1+\alpha\cdot EM)\right)}=T\cdot\frac{\ln 2}{\ln(1+\alpha\cdot EM)}. \tag{27}$$

One should be aware, of course, that these computations do not take into account assimilation and integration processes, intermarriages, etc. Indicators of how fast people of migratory descent may replace those descended from the original population (i.e., in fact, migrated some time prior to the base year) are useful only in stressing the relative importance of migration in the replacement of generations and in helping to indicate the scale of the efforts of integration and adaptation necessary to manage the migration flows observed. More detailed and accurate knowledge about future evolvement of actual population composition may be derived from population projections.

Robustness and projective capabilities of estimates obtained from (27) may be illustrated on data and projections for EU countries available from Eurostat[14]. Fig. 5 presents estimates of time-to-half-replacement based on 2004 estimates for EU countries and regions and also estimates derived from Eurostat projections for the same countries in 2004-2050 (female populations). The latter estimates are derived from comparing Eurostat scenarios with and without migration in 2004-2050:

$$\tau_{0.5}{}^{EST}=(2051-2004)\cdot\frac{\ln(1-0.5)}{\ln\left(B_{2050}^{no\ migr}/B_{2050}\right)}. \tag{28}$$

Here, B_{2050} and $B_{2050}^{no\ migr}$ are births in 2050 within the main and ‘no migration’ scenarios of the Eurostat.

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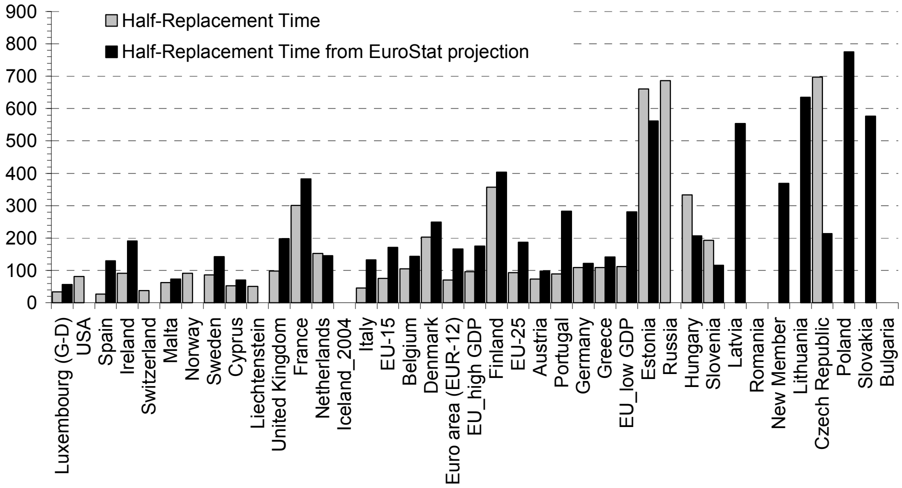
[14] http://epp.eurostat.ec.europa.eu

In general, both estimates are close to each other, although they differ substantially for countries where significant changes in migration are implied in the Eurostat projection. In particular, these were countries with high migration in 2004 (Spain, Ireland, Sweden, UK, Italy and Portugal) where Eurostat assumes migration to fall in the future and also eastern European countries with low net migration where Eurostat assumes net migration to grow before 2050. For the former countries, estimates of THR derived from the 2004 data were shorter, while they were longer for the latter ones, when compared to estimates from the Eurostat projection[15].

Figure 5 Time to half-replacement in EU countries and regions with positive MF estimated from 2004 data and also derived from Eurostat projections for 2004-2050. Females, 2004, years.



Source: Data are estimated from Eurostat projection scenarios (http://epp.eurostat.ec.europa.eu).

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[15] In case of negative EM, THRs are formally indefinite. In such case, they are not shown on the graph.

6. IMPLICATIONS FOR POPULATION AGE COMPOSITION

The regimen of population reproduction through original and migratory births determines the stable state of population age structure just as the traditional reproduction regimen determines the stable state in classical population theory.

The stable population age structure is heavily affected by the Malthusian parameter, otherwise known as Lotka's coefficient. Similarly to the classical case we may introduce the Malthusian parameter as the eventual rate of population change implied by the reproduction regimen. This rate may be derived from the conventional equation relating the Malthusian parameter to survivorship and fertility rates with adding the fertility and survival of migrants into the equation:

$$\sum_x \frac{L_x F_x + L^M{}_x F^m{}_x}{L_0} \lambda^{-x-1} = 1 , \tag{29}$$

where L_x and $L^M{}_x$ are person-years lived by the life table and migrant populations at age x (one-year length intervals are considered; migrants' person-years are presented per one person-year of the indigenous population at the youngest age group)[16].

One might use Eq. (29) if we have estimates for migrants' fertility by age. For practical purposes, however, a simpler approximation may be proposed which is based on the classical concept of generation length:

$$\lambda^T = NR , \tag{30}$$

where T is the generation length and NR is the *Net Reproduction*. We adopt the same concept with substituting the conventional *NR* by *Combined NR*, i.e., adding the migrational component in the same fashion as we did in case

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[16] Note that Eq. (29) explicitly considers fertility of migrants, unlike in other works that considered migration rates as additives to mortality rates, reflected migration in survival rates only, and, thereby, did not address fertility differentials of migrant and non-migrant populations.

of *CF*. In case of low mortality at young ages, one may use the following approximation:

$$CNR \approx \frac{CF}{RF}, \tag{31}$$

where *RF* stands for *Replacement Fertility* (around 2.1).

Implying some guess for the generation length, say, 27 years, one may obtain from (30), (31):

$$\lambda = \sqrt[T]{CNR} \approx \sqrt[T]{\frac{CF}{RF}}\,. \tag{32}$$

Having obtained the Malthusian parameter, one may proceed with the stable age structure. To do so, the easiest way is to note that the Malthusian parameter determines the dynamics of the size of birth cohorts. Therefore, taking into account that indigenous population at age *x* consists of those born *x* years before and who survived to age *x*, we may obtain the age structure of the population born domestically (*including* children of migrants):

$$N^{ab}{}_{x,t} = l(x) N_{0,t-x} = l(x) \lambda^{-x} N_{0,t}\,. \tag{33}$$

Where $l(x) = {}_x p_0$ is the survivorship function, i.e., the accumulated probability of survival from birth to age *x*.

The total population may be derived using proportions to domestic births (5):

$$N_{x,t} = \frac{N_{0,t-x}}{v^{ab}{}_x} = \frac{\lambda^{-x} N_{0,t}}{v^{ab}{}_x} \approx e^{\Delta \sum_{y=0}^{x-\Delta} {}_\Delta m_y} l(x) \lambda^{-x} N_{0,t}\,. \tag{34}$$

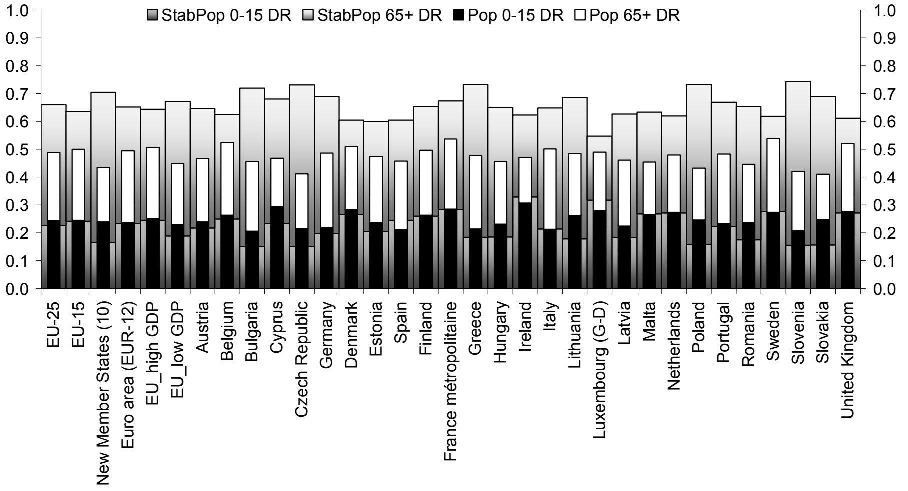
Population born abroad is a difference between (34) and (33):

$$N^{M}{}_{x,t} = N_{x,t} - N^{ab}{}_{x,t} = \left(\frac{1}{v^{ab}{}_x} - l(x) \right) \lambda^{-x} N_{0,t}$$

$$\approx \left(e^{\Delta \sum_{y=0}^{x-\Delta} {}_{\Delta} m_y} - 1 \right) l(x) \lambda^{-x} N_{0,t} \, . \tag{35}$$

From the results presented it is clear that net migration affects the population structure in two ways. Firstly, it affects the Malthusian parameter and shapes the population structure through changing the dynamics of births. Secondly, migrants change the population age structure directly, as they enter the population at ages distributed throughout the whole life span.

Figure 6 Dependency ratios for both actual and stable equivalent populations derived from 2004 data for EU countries (stable populations are derived using CF in 2004).



Source: Data are estimated from Eurostat projection scenarios (http://epp.eurostat.ec.europa.eu).

Usage of the relations obtained is illustrated on Figs. 6 and 7. Fig. 6 presents estimates of dependency ratios for both actual and stable equivalent populations derived from 2004 data for EU countries. The next figure shows these ratios derived from projected results by Eurostat for 2050. These figures demonstrate that the age structure of EU populations does converge to that predicted from a stable equivalent population derived from CF-based Malthusian parameter and net migration profile. This convergence may also be observed from the dynamics of dependency rates calculated for each year for separate countries or regions. Fig. 8 presents such dynamics for EU-25 as a whole. This figure may be compared to Fig. 9 which presents the same dynamics using a traditional stable equivalent population instead of the stable population derived from the combined effect of fertility and migration. One may note that a conventional stable equivalent population results in significantly biased estimates for asymptotic stable structure of the population[17]. Indeed, migration makes younger the population age structure, as it is supposed by stable population structures presented above and also is reflected in projection by Eurostat. Analysis of population ageing prospects without taking migratory impact into count, on the contrary, may result in biased conclusions.

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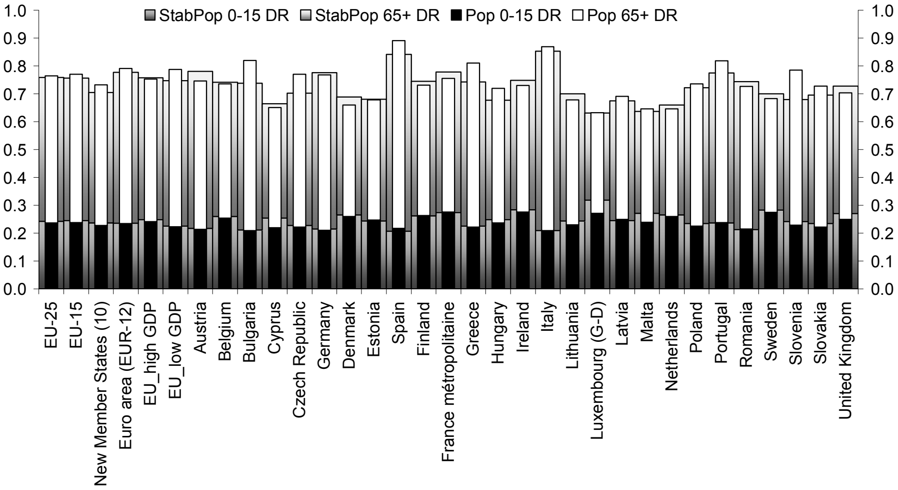
[17] This may also have important implications for the momentum concept, which we do not address here.

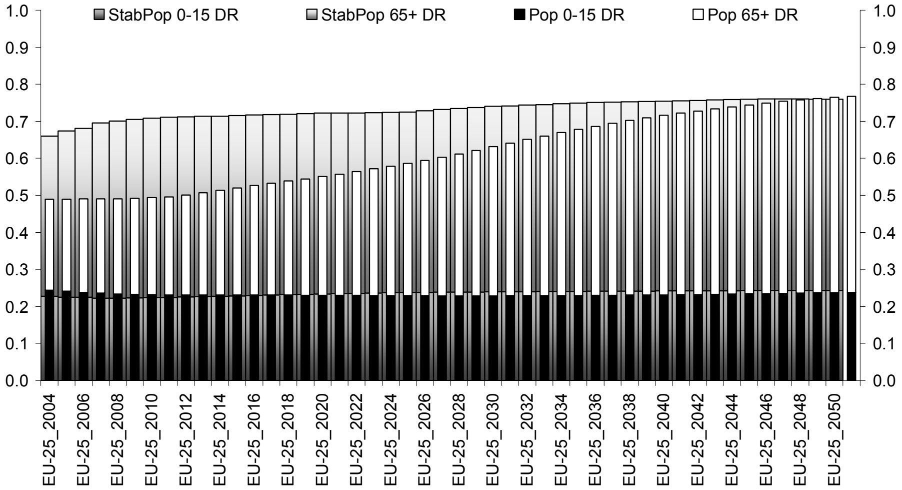
Figure 7 Dependency ratios for both actual and stable equivalent populations derived from Eurostat projection for EU countries by 2050 (stable populations are derived using CF in 2050).



Source: Data are estimated from Eurostat projection scenarios (http://epp.eurostat.ec.europa.eu).

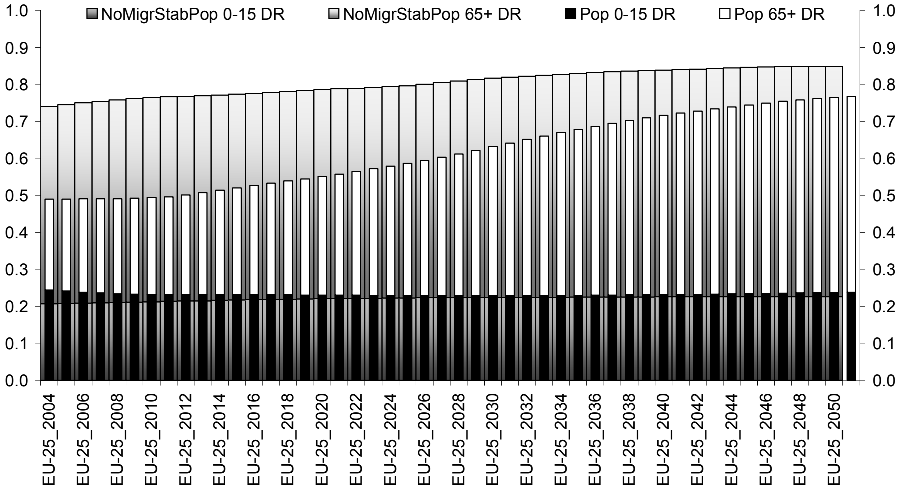
Comparison of Figs. 8 and 9 also points to the fact that it is increased longevity that will play the main role in population ageing in Europe, which explains why migration was shown not to be effective in precluding the population ageing. This does not undermine, however, the role of migration in reproducing the birth cohorts and in supporting the labour force in receiving countries.

Figure 8 Dynamics of dependency ratios for both actual and stable equivalent populations derived for EU-25 from Eurostat projection for 2004-2050 (stable populations are derived using CF in corresponding years).



Source: Data are estimated from Eurostat projection scenarios (http://epp.eurostat.ec.europa.eu).

Figure 9 Dynamics of dependency ratios for both actual and stable equivalent populations derived for EU-25 from Eurostat projection for 2004-2050 (stable populations are derived using conventional relations ignoring migration).



Source: Data are estimated from Eurostat projection scenarios (http://epp.eurostat.ec.europa.eu).

7. CONCLUSION

The incorporation of migration into population models as a factor of reproduction rather than as a simple addition to natural increase improves our understanding of the role of migration in the life experience of cohorts, in the reproduction of populations and in the evolution of the components of the population by origin. It is also relevant to studying the evolution of the population's age-structure.

To reflect the role of migration in the life experience of cohorts, we introduce the concept of 'Completed Net Migration' as the number of net migrants expected to join the original birth cohort throughout its life span. Direct computation requires inputs in the form of migration flows per unit of the receiving population's birth cohort. Usually, however, data are available in terms of age-specific migration rates. The paper proposes a method for deriving the former rates from the latter based on the assumption of constant age-specific net migration rates.

In the context of population reproduction we introduce the concept of 'Migratory Fertility' as a measure of migrants' addition to the reproduction of the birth cohorts of the population itself. Again, estimation procedures depend on assumptions about the nature of migration flows. Based on a comparative examination of several approaches, robust estimating procedures are suggested which reflect the differential fertility of migrants and also significant links between migration at different ages.

The reproduction regimen of the population at large is characterised by the 'Combined Fertility', which is approximated by the sum of conventional Total Fertility and of Migratory Fertility. The concept of a stable equivalent population is also adjusted to include the impact of migration on population growth and structure. The revision of the stable equivalent population concept provides a basis for studying the prospects of population age composition and ageing.

Our indicators do not reflect the compositional effects of actual migration, varying in time and affecting the population that is a mix of cohorts[18]. Just like the *TF* which reflects the fertility level of a given cohort only, the *CF* and *MF* reflect the replacement schedule of a specific population cohort only. They predict the future of the population only if migration and fertility rates remain constant at observed levels. Our measures transparently reflect the unavoidable consequences of the observed rates. This differs from the simulation approach which provides no insights into the mechanics of the evolution of population composition. The concepts presented in this paper can be developed further. In particular, rates may be derived for real birth cohorts as well as for synthetic calendar cohorts.

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[18] An example of how such an analysis might be conducted is given in the Appendix.

Techniques proposed in the literature for addressing the effect of fertility postponement on population dynamics may also be important for the framework proposed here. For the purposes of fertility policies it might also be important to investigate migratory effects on births by parity. In all these cases, however, more complicated models might be needed.

APPENDIX. CASE STUDY: IMPLICATIONS OF MIGRATION RATES IN AUSTRIA IN 2004

This case study is based on migration data for 2004 available from Statistics Austria (2005). Importantly, these data differ substantially from migration assumptions implied for Austria in the same year in Eurostat projections. Eurostat projections imply females' net migration to Austria at about 14300 in 2004, while the official data for the same year gave 25784, i.e., almost twice as much. Therefore, the figures given below, which are based on official estimates from Statistics Austria, are not consistent with the EU comparisons given above, which were based on EU scenarios. It must also be noted that 2004 was a year of exceptionally high net migration to Austria.

Another important difference from the results presented above is that we differentiate migration by citizenship. We decompose the Migratory Fertility into components of an Austrian migrants' MF and a non-Austrian migrants' MF and suppose the fertility of Austrian migrants to be identical to that of general population. *We do not address the naturalisation and assimilation processes, so our estimates should not be taken at face value as predictors of future social turnovers in Austria.* Non-Austrian migrants' fertility is supposed to be 50% higher than that of non-migrants. This assumption is based on data on *TF* by migrant status for Austria [Statistics Austria 2006: 783] and other EU countries [Coleman 2006: 407]. Results for 2004 are presented in Table 1. In an alternative scenario we suppose only a 10% excess of non-Austrian migrants' *TF*, see the results in Table 2.

In addition to calculating the *CF* and its components, we calculate two variants of time duration to replacing half the population by descendants of *non-Austrian* migrants. First, we calculate this time for rates observed in 2004. Secondly, we calculate this duration for a scenario in which net migration of non-Austrians rises/drops to the level sufficient to support simple replacement of population cohorts under the fertility and Austrians' net migration rates fixed at levels observed in 2004. For these calculations the simple replacement fertility was set to be 2.1 for females and 2.0 for males. The latter time durations illustrate the possible impact of replacement immigration, i.e., of the immigration sufficient to prevent depopulation.

Vienna—having a *TF* at the national average level—is fed by the highest net immigration rates of non-Austrians. As a result, the *Combined Fertility* of Vienna is much higher than the simple replacement level for both males and females, of which only less than half is the share of Austrians (*TF* plus *MF* of Austrians). The combination of average fertility, moderate immigration of Austrians and high immigration of non-Austrians results in a dramatic net impact of migration on the population of Vienna in the future. Under population replacement rates fixed at 2004 levels it will take only about 10-20 years before half of the dynamically growing population of Viennese children will be descendants of those who immigrated after 2004. Although our indicators reflect the impact on future *birth* cohorts, it is safe to conclude that the share of net migrants in the mid-aged population will roughly be close to that of their children (one should not forget, however, to take account of different fertility levels; i.e., the migrants' share among parents will be lower than that of their children).

Four other regions (Lower Austria, Upper Austria, Tyrol and—most of all—Vorarlberg) also have the *CF* close to or above the simple replacement level. These regions—assuming the continuation of rates observed in 2004—do not seem to face dramatic depopulation. Of course, these benefits are in a remarkable part due to net migration. Vorarlberg shows negative a net impact on population replacement from migration of Austrians, which is more than compensated by net immigration of non-Austrians. Although this region has the highest fertility rate, the share of Austrians in replacing its population is close to or even lower than that for the Austria as a whole. Lower Austria, on the contrary, has only an average fertility level. But this is supplied by remarkable impact of positive net migration of Austrians and—as a result—this region is among those with highest share of Austrians in replacing its birth cohorts.

Table A1 *Combined Fertility* of Austrian population by federal lands and its components in 2004 with durations of time to replacing half of the population by descendants of non-Austrian migrants. Non-Austrian migrants' *TF* is set to be 50% higher than that of Austrians.

Land	**CF**	**TF**	**MF** (Austrians)	**MF** (non-Austrians)	**THR** (2004 rates)	**THR** (simple total replacement)
			Males			
Austria	**2.33**	**1.31**	**-0.11**	**1.12**	**31 year(s)**	**40 year(s)**
Burgenland	**1.44**	1.14	-0.02	0.32	79 year(s)	35 year(s)
Carinthia	**1.26**	1.22	-0.26	0.30	75 year(s)	28 year(s)
Lower Austria	**2.08**	1.32	0.13	0.63	56 year(s)	63 year(s)
Upper Austria	**1.81**	1.37	-0.20	0.63	47 year(s)	38 year(s)
Salzburg	**1.86**	1.35	-0.21	0.72	41 year(s)	36 year(s)
Styria	**1.61**	1.17	-0.15	0.58	45 year(s)	30 year(s)
Tyrol	**1.90**	1.33	-0.06	0.62	51 year(s)	45 year(s)
Vorarlberg	**2.03**	1.48	-0.26	0.80	40 year(s)	41 year(s)
Vienna	**6.28**	1.36	-0.24	5.16	12 year(s)	35 year(s)
			Females			
Austria	**2.56**	**1.42**	**-0.11**	**1.25**	**28 year(s)**	**40 year(s)**
Burgenland	**1.80**	1.27	-0.10	0.63	43 year(s)	32 year(s)
Carinthia	**1.49**	1.35	-0.30	0.44	54 year(s)	27 year(s)
Lower Austria	**2.27**	1.45	0.15	0.66	54 year(s)	69 year(s)
Upper Austria	**2.06**	1.50	-0.23	0.79	39 year(s)	37 year(s)
Salzburg	**1.95**	1.42	-0.21	0.74	39 year(s)	34 year(s)
Styria	**1.88**	1.31	-0.17	0.73	38 year(s)	31 year(s)
Tyrol	**2.15**	1.42	0.01	0.71	46 year(s)	49 year(s)
Vorarlberg	**2.34**	1.56	-0.32	1.09	30 year(s)	36 year(s)
Vienna	**6.31**	1.42	-0.25	5.14	11 year(s)	32 year(s)

Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.

The rest of Austria (especially Burgenland and Carinthia) will face a population decline and ageing under rates fixed at 2004 levels. Even the immigration observed in 2004, which is not enough to prevent depopulation and ageing of these regions, will produce more than half of descendant population in about 50-100 years. Were immigration to rise up to the level sufficient for simple replacement, this would take only about 30-60 years, i.e. one to two generations.

Table A2 *Combined Fertility* of Austrian population by federal lands and its components in 2004 with durations of time to replacing half of the population by descendants of non-Austrian migrants. Non-Austrian migrants' *TF* is set to be 10% higher than that of Austrians.

Land	**CF**	**TF**	**MF** (Austrians)	**MF** (non-Austrians)	**THR** (2004 rates)	**THR** (simple total replacement)
Males						
Austria	**2.03**	**1.31**	**-0.11**	**0.82**	**39 year(s)**	**40 year(s)**
Burgenland	**1.36**	1.14	-0.02	0.24	104 year(s)	35 year(s)
Carinthia	**1.18**	1.22	-0.26	0.22	99 year(s)	28 year(s)
Lower Austria	**1.92**	1.32	0.13	0.46	73 year(s)	63 year(s)
Upper Austria	**1.64**	1.37	-0.20	0.46	60 year(s)	38 year(s)
Salzburg	**1.67**	1.35	-0.21	0.53	53 year(s)	36 year(s)
Styria	**1.45**	1.17	-0.15	0.42	58 year(s)	30 year(s)
Tyrol	**1.73**	1.33	-0.06	0.46	66 year(s)	45 year(s)
Vorarlberg	**1.81**	1.48	-0.26	0.59	51 year(s)	41 year(s)
Vienna	**4.90**	1.36	-0.24	3.78	14 year(s)	35 year(s)
Females						
Austria	**2.23**	**1.42**	**-0.11**	**0.92**	**35 year(s)**	**40 year(s)**
Burgenland	**1.63**	1.27	-0.10	0.46	56 year(s)	32 year(s)
Carinthia	**1.37**	1.35	-0.30	0.32	70 year(s)	27 year(s)
Lower Austria	**2.09**	1.45	0.15	0.49	71 year(s)	69 year(s)
Upper Austria	**1.85**	1.50	-0.23	0.58	50 year(s)	37 year(s)
Salzburg	**1.75**	1.42	-0.21	0.54	50 year(s)	34 year(s)
Styria	**1.68**	1.31	-0.17	0.54	49 year(s)	31 year(s)
Tyrol	**1.95**	1.42	0.01	0.52	60 year(s)	49 year(s)
Vorarlberg	**2.05**	1.56	-0.32	0.80	38 year(s)	36 year(s)
Vienna	**4.94**	1.42	-0.25	3.77	13 year(s)	32 year(s)

Statistics Austria. Demographisches Jahrbuch 2004. Vienna, 2005.

On average, the Austrian *CF* was around or above the simple replacement level in 2004. Under rates observed in 2004 it is likely to expect relative stability or growth of birth cohort sizes in the Austrian future with ageing of population due to life expectancy improvements only. About one-third of the *CF*, however, is due to net immigration of non-Austrians. Therefore, in about 40-50 years half the births in Austria will be descendants from foreigners who immigrated after 2004 if the observed rates do not change in the future.

Actual replacement of generations in Austria will depend on the future developments in migration. Some estimates may be derived, however, from our estimates on *CF* and *MF*. It might be interesting to look into the impact of immigration including the years preceding 2004. This could be based on retrospective statistics.

One should note, however, that the idea of counting migrants' descendants may become meaningless if being taken to its extreme. Any population that is open to even a moderate immigration will eventually consist of immigrants' descendants. It is only a matter of time to replace half or any other share of the population by migrant descendants. The only exception from this migrational ergodic trap is for the entire population of mankind, which has no influence of migration. A huge population size may also guarantee that the share of 'indigenous' population and of 'returning' migrants, who are the descendants of those who previously *emigrated* from the same population will never become negligible. A more realistic approach would be to look into the impact of net migration during, for instance, the last generation-long period, i.e., during the past 25-30 years.

Using the population balance estimates for Austria [Statistics Austria 2005] since 1981, one may estimate for example that the total sum of net migration rates of non-Austrians over all the period was eight times higher compared to the figure for 2004 alone. For Austrians this ratio was about 14. Our estimates for Effective Migration (averages for both sexes) were -0.08 for Austrians and 0.58 for non-Austrian migrants in 2004. Therefore, we may conclude roughly that net immigration of Austrians resulted in EM of $14 \cdot (-0.08) = -1.10$ over 24 years, i.e., at an average of -0.05. Net migration of non-Austrians was at EM of $8 \cdot 0.58 = 4.64$ with average 0.19. Therefore, during 21 years indigenous population's share in population replacement was being reduced by a factor about $(1-0.05)/(1-0.05+0.19) \approx 0.83$ per generation. With one generation being 28 years this means that the posterity of Austrian indigenous inhabitants as of 1981 will constitute the share $0.83^{21/28} \approx 0.87$, i.e. 87%, in the future population of Austria under no net migration flows after 2004.

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